Plato Hare & Hound 2
Modelling and Acoustic Glitches

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Stellar modelling

Evolutionary code used: MESA (Paxton et al. 2011, 2013)

- OPAL EoS (Rogers & Nayfonov 2002)
- OP opacity (Badnell 2005, Seaton 2005)
- Ferguson low T opacity (Ferguson et a. 2005)
- GS98 mixture (Grevesse & Sauval 1998)
- NACRE reaction rates (Angulo et al. 1999)
- Standard MLT (Cox & Giuli 1968)
- Core overshoot (Herwig 2000)
- Diffusion of helium and heavy elements (Thoul et al. 1994)

Model frequencies calculated using ADIPLS (Christensen-Dalsgaard 1991)

Stellar modelling

- Grid of stellar models with random values of M, Y_0, Z_0, α, $d_{\rm ov}$:

	$M/M_\odot$	Y_0	$[\mathrm{Fe/H}]_i$	α	$d_{\rm ov}$
HH2a	1.00 – 1.30	0.22 – 0.32	0.05 – 0.35	1.5 – 2.0	0.00 – 0.03
HH2b	1.00 – 1.30	0.22 – 0.32	0.00 – 0.20	1.5 – 2.0	0.00 – 0.03

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- 2000 evolutionary tracks for each star
- Model frequencies are corrected for surface effect (Kjeldsen et al. 2008)
- On each track find best model by minimising

$$\chi^2_\nu = \frac{1}{N}\sum_i \left(\frac{\nu_{\rm obs} - \nu_{\rm mod}}{\sigma_{\rm obs}}\right)^2_i$$

Stellar modelling

- Final model chosen by minimising

$$\chi^2 = \sum_q \frac{(q_{\rm mod} - q_{\rm obs})^2}{\sigma_q^2}$$

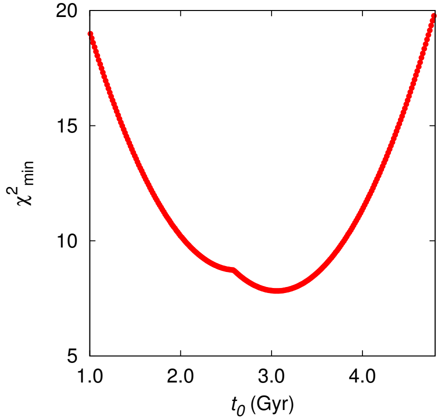
where q is $\{T_{\rm eff}, L, [{\rm Fe/H}]_s, \langle\Delta_0\rangle, \langle r_{02}\rangle, r_{01}(n), r_{01}(n+3)\}$, n being a suitably chosen radial order.

- The choice of $r_{01}(n+3)$ was made to avoid correlation with $r_{01}(n)$.

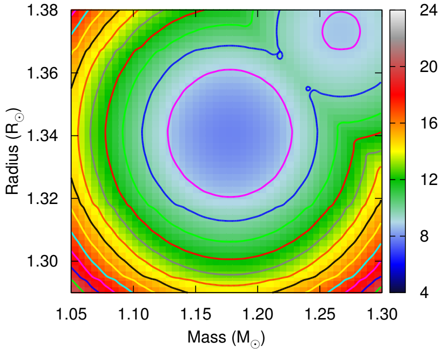
- Similar weightage to non-seismic and seismic parameters.

HH2a

Age

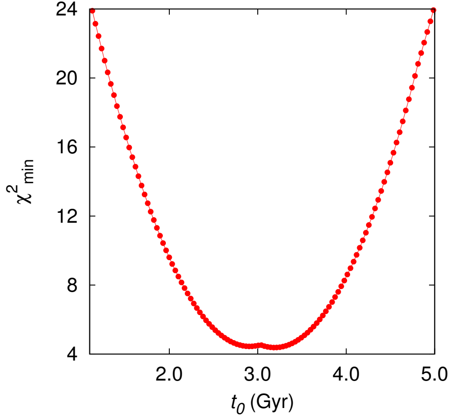


Mass & Radius



HH2b

Age



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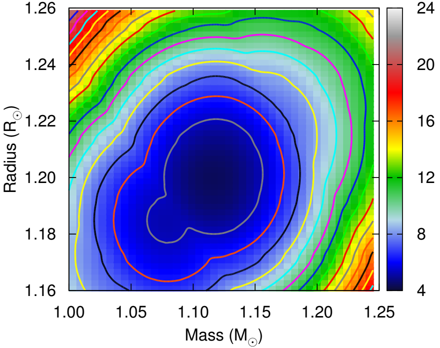

Modelling results

Parameter	HH2a	HH2b
$M/M_\odot$	1.18 ± 0.05	1.11 ± 0.04
$R/R_\odot$	1.34 ± 0.02	1.20 ± 0.02
Age(Gyr)	3.0 ± 0.5	3.1 ± 0.4
Y_s	0.240	0.239
Y_0	0.290	0.277
$[Fe/H]_s$	0.085	-0.015
$\alpha_{\rm MLT}$	1.534	1.649
$R_{\rm BCC}/R_\odot$	0.081	0.000
$R_{\rm BCZ}/R_\odot$	1.086	0.946

Acoustic Glitches

Two techniques for glitch fitting

- **A :** Fit functional form to frequencies for BCZ, He and smooth components simultaneously.

Acoustic Glitches

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- **A :** Fit functional form to frequencies for BCZ, He and smooth components simultaneously.
- **B :** Fit functional form to second differences for BCZ, He and smooth components simultaneously.

Method A (Verma & Antia)

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- Uncertainties determined through MC realisations

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- Fit $\delta^2\nu$ to a suitable function representing the oscillatory signals from the two acoustic glitches:

$$\begin{aligned}\delta^2\nu &= \text{Smooth component}\\ &\quad + \text{BCZ component}\\ &\quad + \text{HeII component}\end{aligned}$$

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$$\begin{aligned}\delta^2\nu \;=\;& a_0 + a_1\nu \\ & + (b_2/\nu^2)\sin(4\pi\nu\tau_{\mathrm{BCZ}} + 2\phi_{\mathrm{BCZ}}) \\ & + (c_0\nu\exp(-c_2\nu^2))\sin(4\pi\nu\tau_{\mathrm{II}} + 2\phi_{\mathrm{II}})\end{aligned}$$

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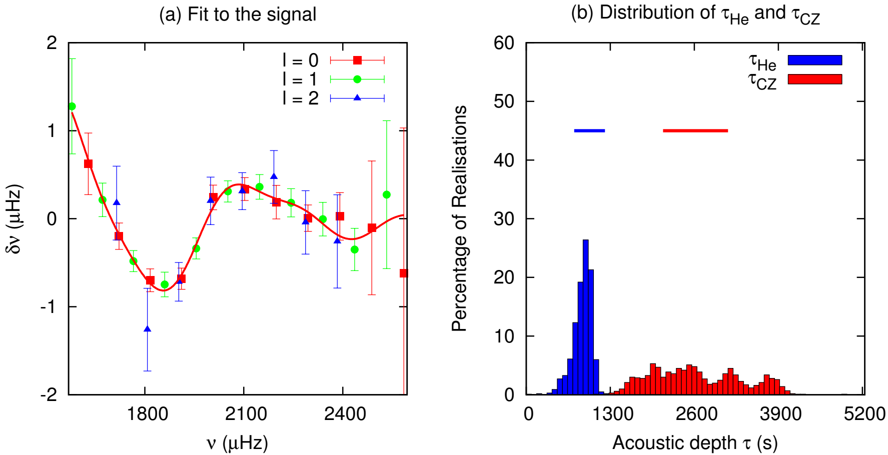
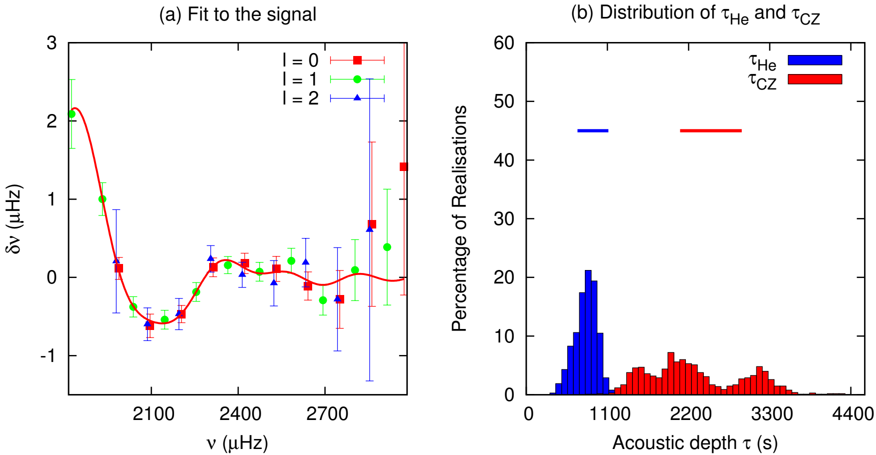
- Number of free parameters: 9
- Uncertainties determined through MC realisations

Glitch results — Method A

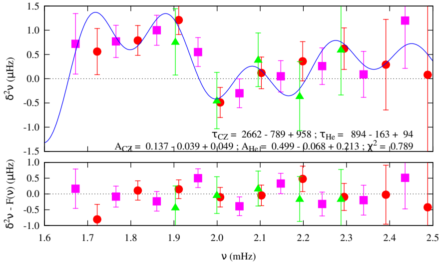
HH2a

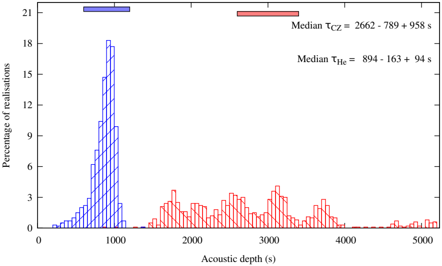


HH2b

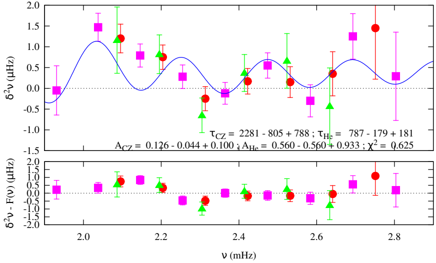

(a) Fit to the signal
3
2
1
0
-1
δν (μHz)
2100
2400
2700
ν (μHz)
l = 0
l = 1
l = 2
(b) Distribution of τHe and τCZ
60
50
40
30
20
10
0
Percentage of Realisations
0
1100
2200
3300
4400
Acoustic depth τ (s)
τHe
τCZ

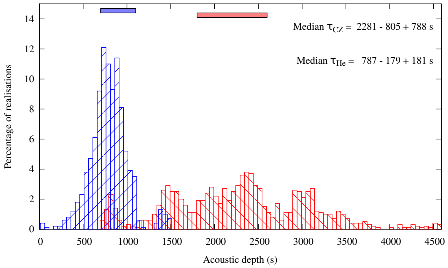
HH2a


τ_{CZ} = 2662 - 789 + 958 ; τ_{He} = 894 - 163 + 94
A_{CZ} = 0.137 - 0.039 + 0.049 ; A_{He} = 0.499 - 0.068 + 0.213 ; χ^2 = 0.789
$\delta^2\nu$ (μHz)
$\delta^2\nu$ - F(ν) (μHz)
ν (mHz)


Median τ_{CZ} = 2662 - 789 + 958 s
Median τ_{He} = 894 - 163 + 94 s
Percentage of realisations
Acoustic depth (s)

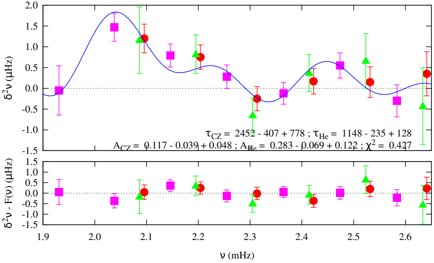
HH2b — Full range

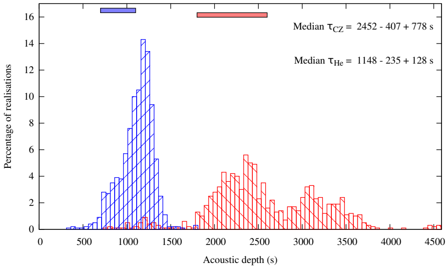




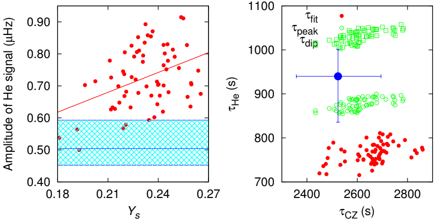
Glitch results — Method B

HH2b — Restricted range



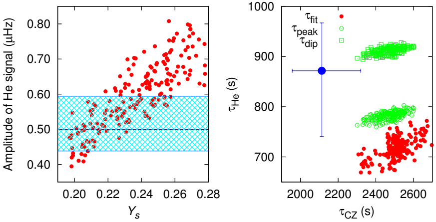


HH2a



Amplitude of He signal (μHz)
0.90
0.80
0.70
0.60
0.50
0.40
0.18
0.21
0.24
0.27
Y_s
1100
1000
900
800
700
τ_{fit}
τ_{peak}
τ_{dip}
τ_{He} (s)
2400
2600
2800
τ_{CZ} (s)

HH2b



Amplitude of He signal (μHz)
0.80
0.70
0.60
0.50
0.40
0.20
0.22
0.24
0.26
0.28
Y_s
1000
900
800
700
τ_{fit}
τ_{peak}
τ_{dip}
τ_{He} (s)
2000
2200
2400
2600
τ_{CZ} (s)

Glitch results

Parameter	HH2a	HH2b
$\tau_{0,\mathrm{mod}}$(s)	5255	4566
$\tau_{\mathrm{CZ,obs}}$(s) (A)	2520 ± 300	2110 ± 300
$\tau_{\mathrm{CZ,obs}}$(s) (B)	2662 ± 300	2281 ± 350
$\tau_{\mathrm{CZ,mod}}$(s)	2690	2520
$\tau^{c}_{\mathrm{CZ,mod}}$(s)	2678	2444
$\tau_{\mathrm{He,obs}}$(s) (A)	940 ± 120	870 ± 150
$\tau_{\mathrm{He,obs}}$(s) (B)	894 ± 130	787 ± 180
$\tau_{\mathrm{He,mod}}$(s)	764	712
Y_s (from glitch)	$\cdots$	0.21 ± 0.03
Y_0 (from glitch)	$\cdots$	0.25 ± 0.03
Y_s (from model)	0.240	0.239
Y_0 (from model)	0.290	0.277