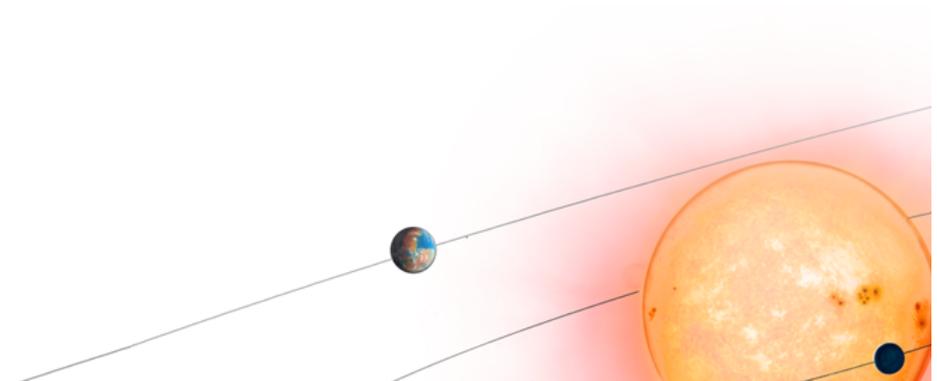
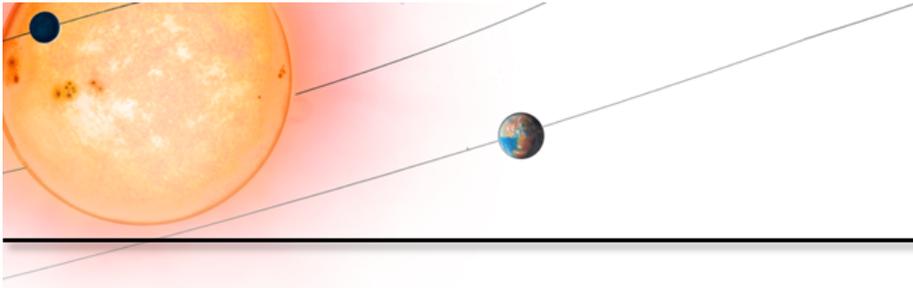


Challenges for fitting observations of PLATO subgiant stars

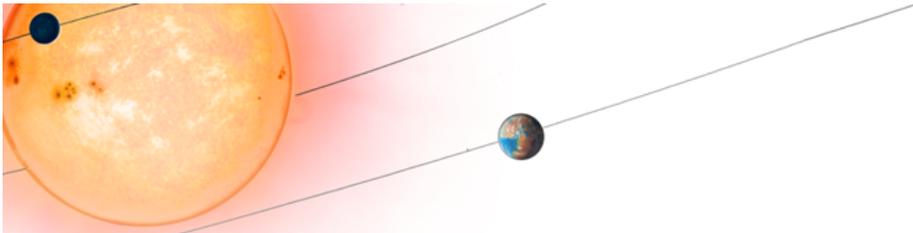
PLATO STESCI Workshop – Milazzo – 05/2018
Sébastien Deheuvels





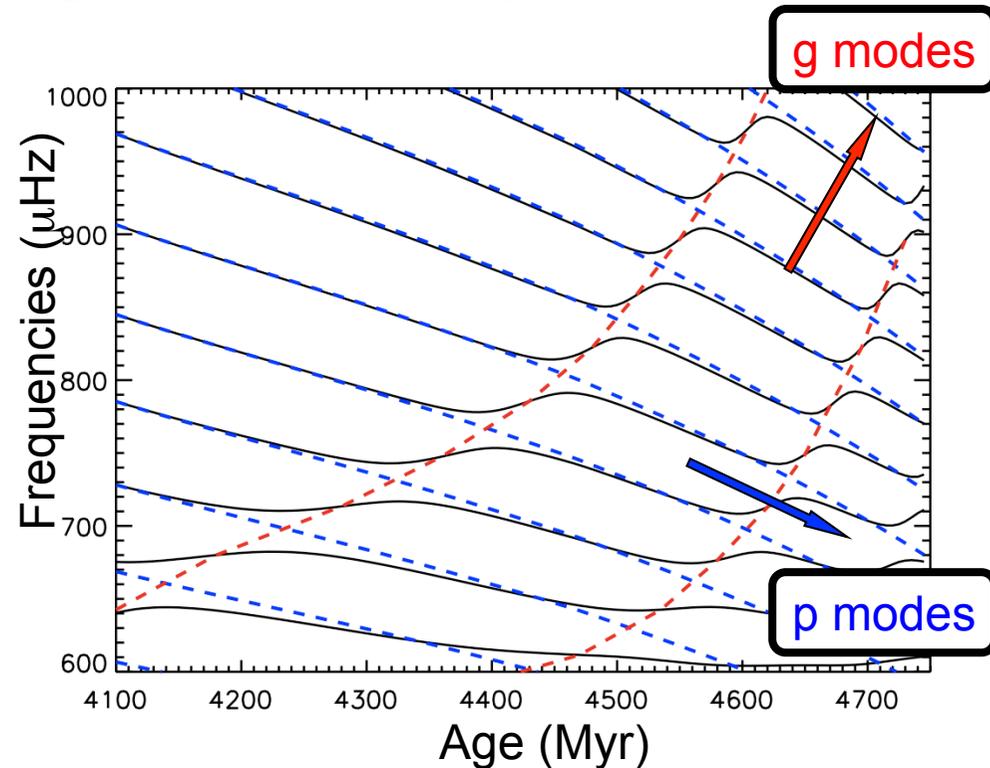
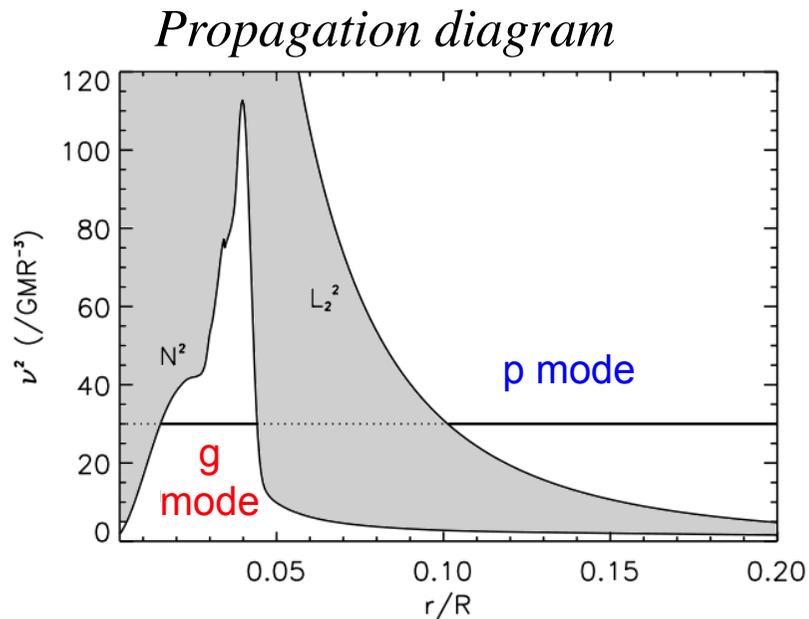
Outline

- What can mixed modes tell us about stellar cores?
- Current modeling techniques and performances
- Challenges for stellar modeling

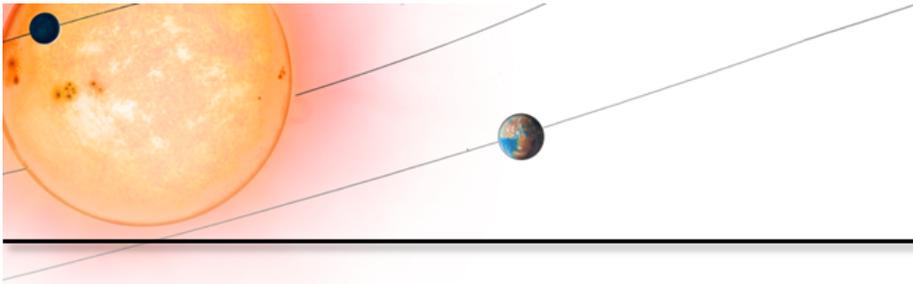


Mixed modes

- For evolved stars (past main-sequence turnoff), the Brunt-Väisälä frequency becomes huge (central density $\rho_c \nearrow \nearrow$)

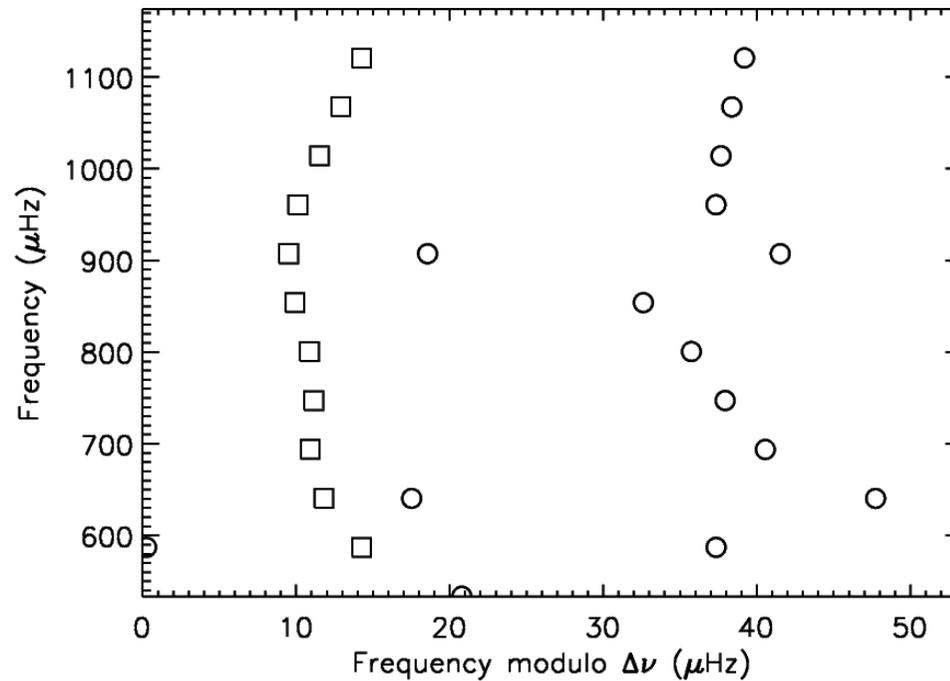


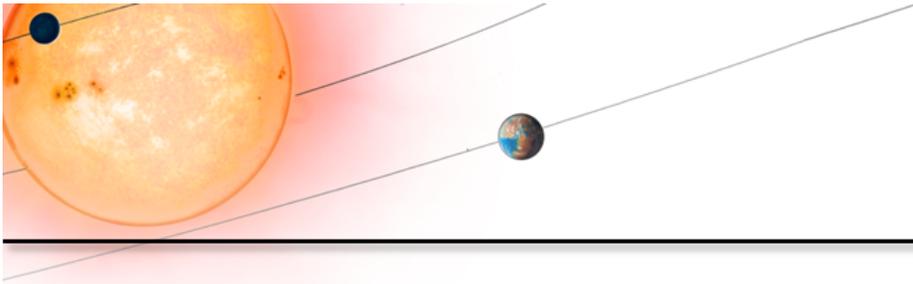
- Interest of mixed modes :
 1. They are sensitive to the core structure
 2. Their amplitudes are much larger than those of pure g modes



Oscillation spectra of subgiants

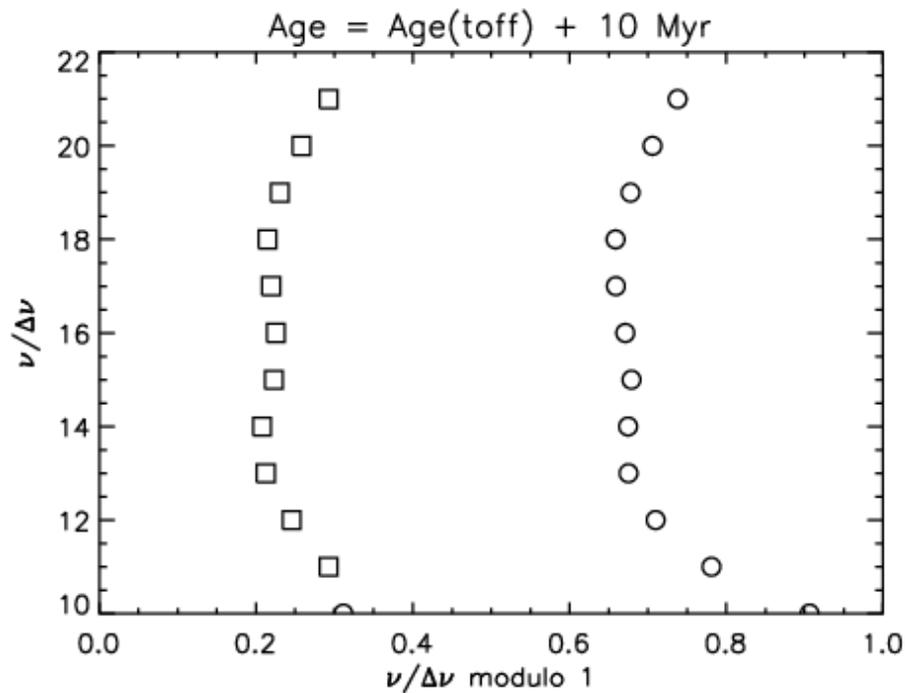
- Variations in the oscillation spectrum during the subgiant phase
 - Example for a $1.3-M_{\odot}$ model



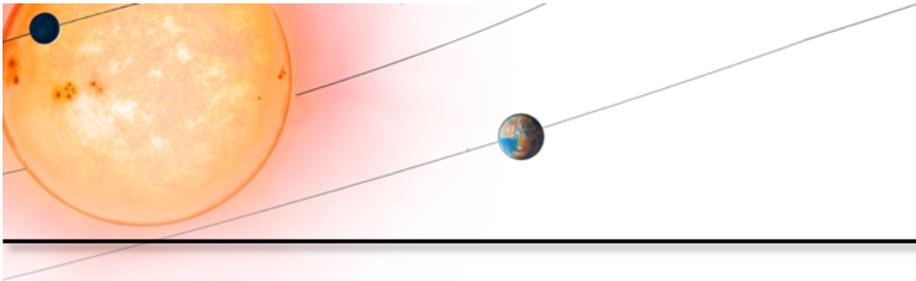


Oscillation spectra of subgiants

- Variations in the oscillation spectrum during the subgiant phase
 - Example for a $1.3-M_{\odot}$ model

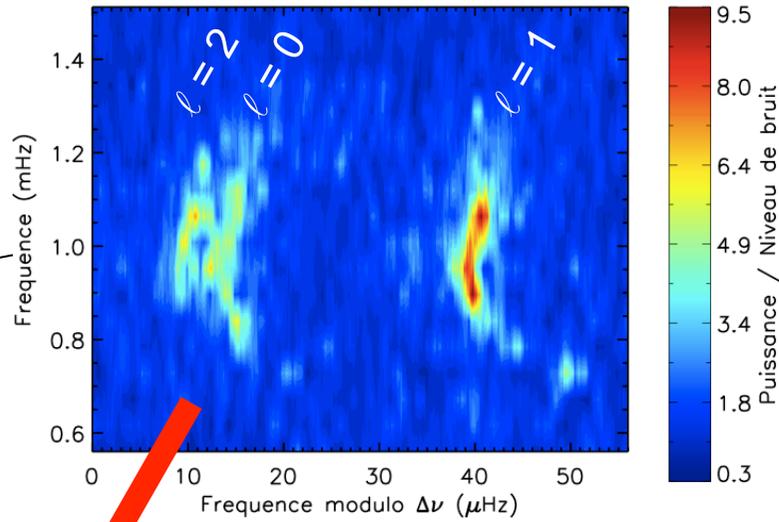


- 1st avoided crossing
- 2nd avoided crossing
- 3rd avoided crossing
- 4th avoided crossing

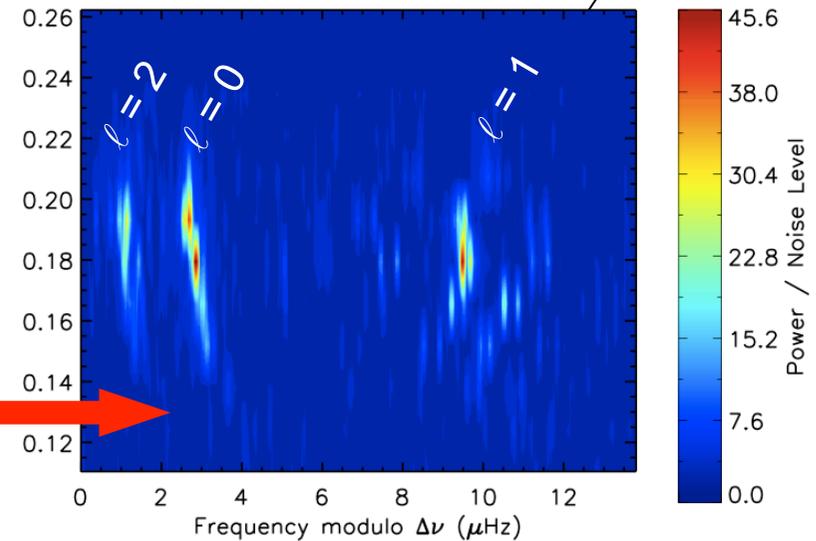
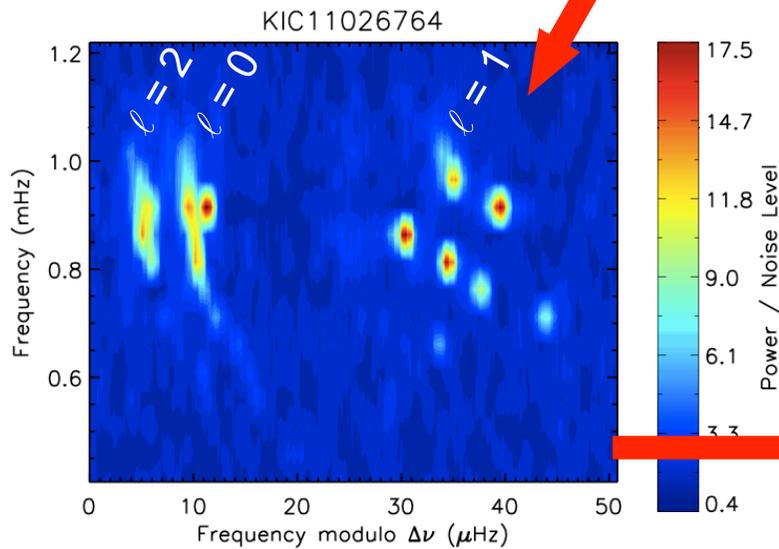


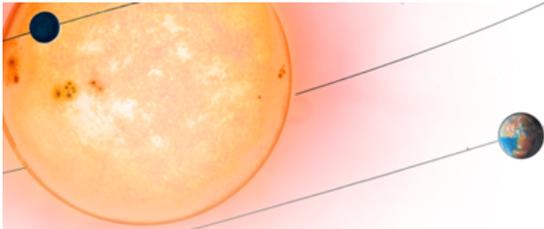
Oscillation spectra of subgiants

Subgiants



Base of the RGB





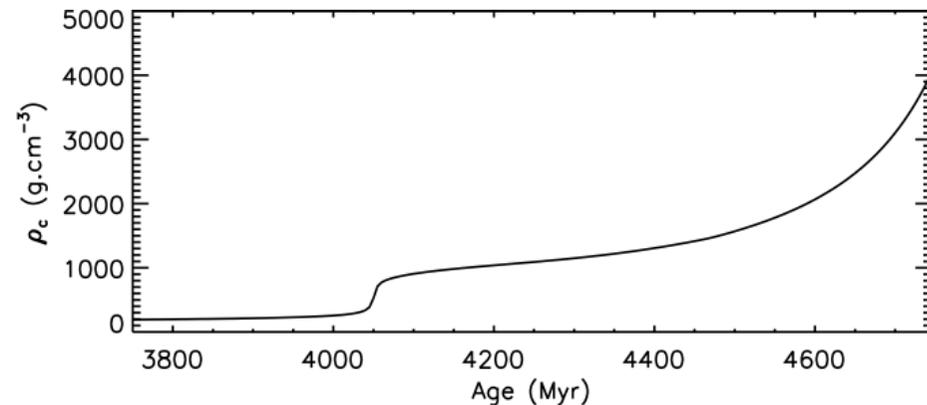
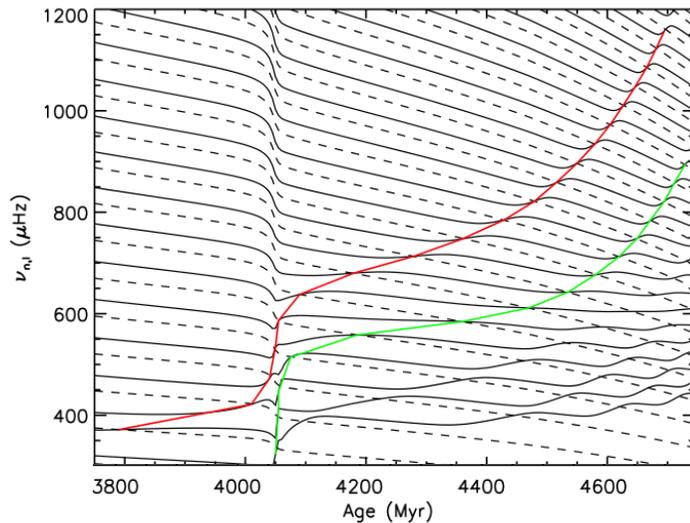
Interest of mixed modes

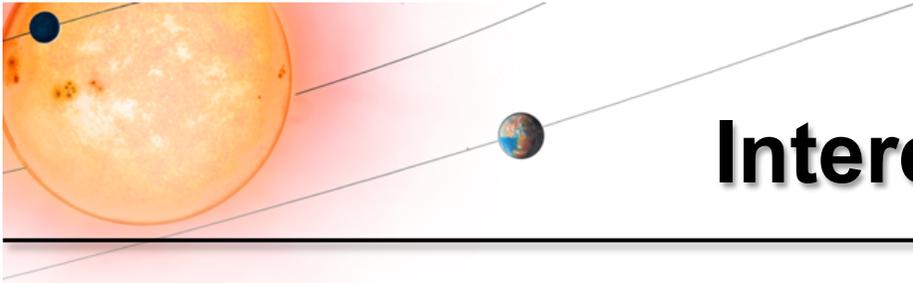
- G-modes sensitive to the **Brunt-Väisälä profile in the core**

$$\omega_g \sim \int_{r_i}^{r_o} \frac{N}{r} dr$$

- Variations in the frequency of g modes tightly related to the variations in ρ_c

⇒ **Strong constraints on the age** (for given input physics)

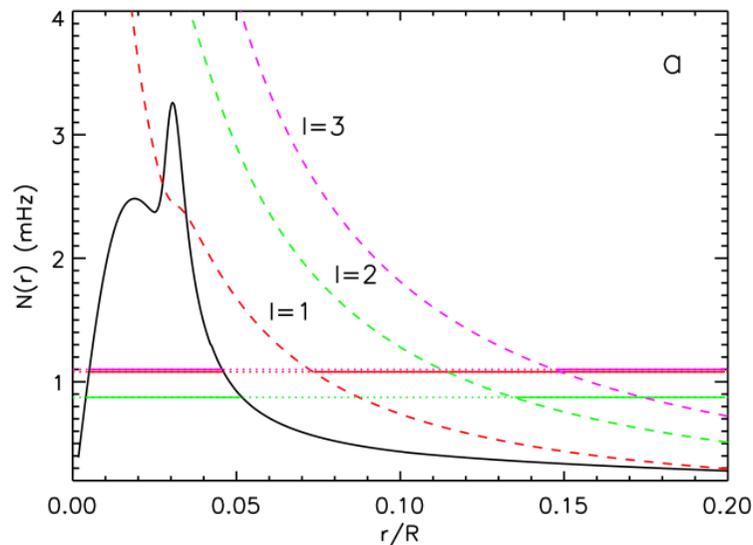




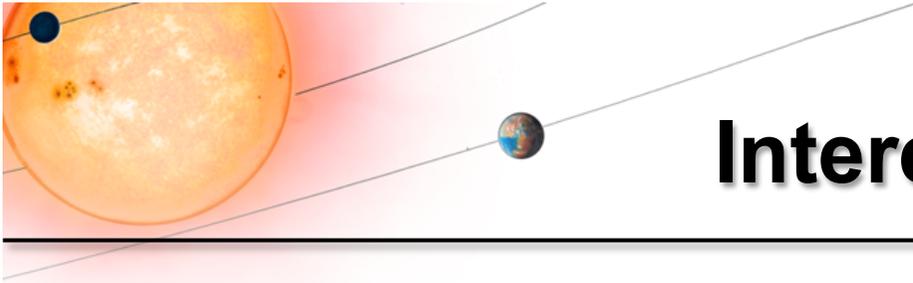
Interest of mixed modes

- G-modes sensitive to the **structure of the evanescent zone** between the p-mode and g-mode cavities
- JWKB approximation: coupling between p-mode and g-mode cavities given by

$$q \approx \frac{1}{4} \exp \left(-2 \int_{\text{ZE}} \kappa \, dr \right) \quad \text{with} \quad \kappa^2 \approx \frac{\omega^2}{c^2} \left(\frac{S_l^2}{\omega^2} - 1 \right) \left(1 - \frac{N^2}{\omega^2} \right)$$

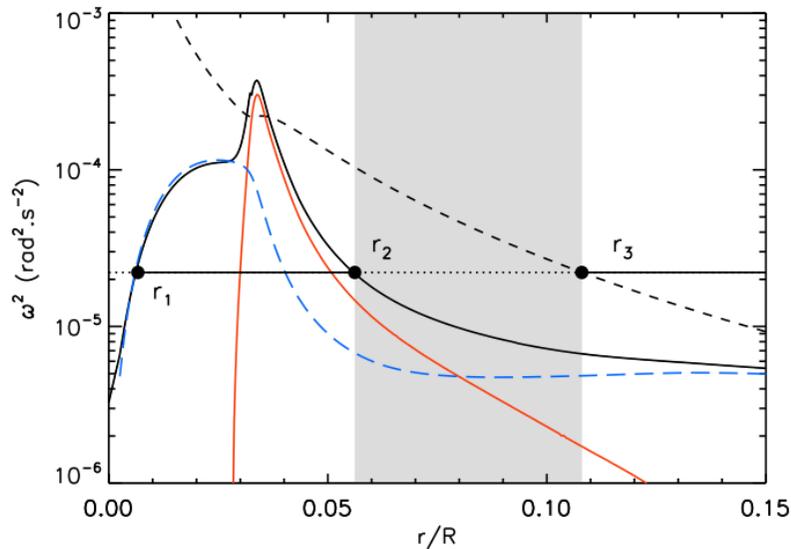


- Coupling decreases for increasing degree ℓ



Interest of mixed modes

- What is mode coupling $q \approx \frac{1}{4} \exp\left(-2 \int_{\text{ZE}} \kappa dr\right)$ sensitive to?

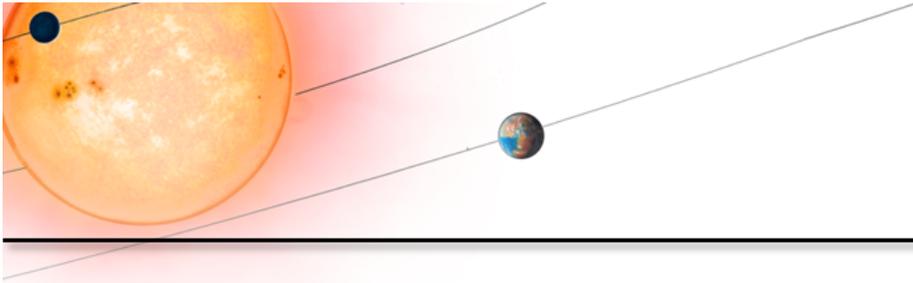


$$N^2 = \frac{g}{H_P} (\nabla_{\text{ad}} - \nabla) + \frac{g}{H_P} \nabla_{\mu}$$

- Evanescent zone essentially sensitive to ∇_{μ}
- ⇒ **Sensitivity to mixing processes**

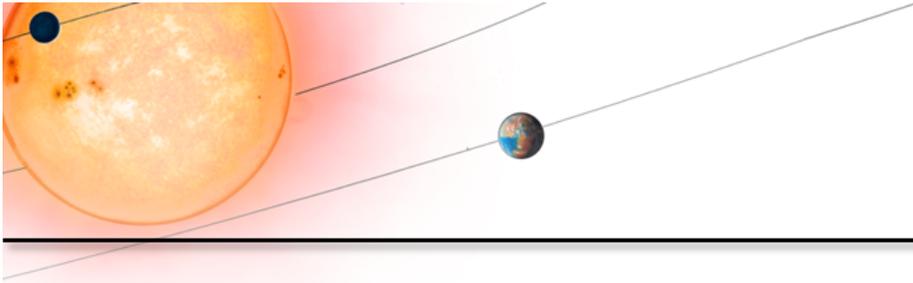
- Good news to test stellar physics!
- Not so good news to measure masses, radii, ages, which thus depend on the modeling of mixing processes

+ Sensitivity to stellar mass



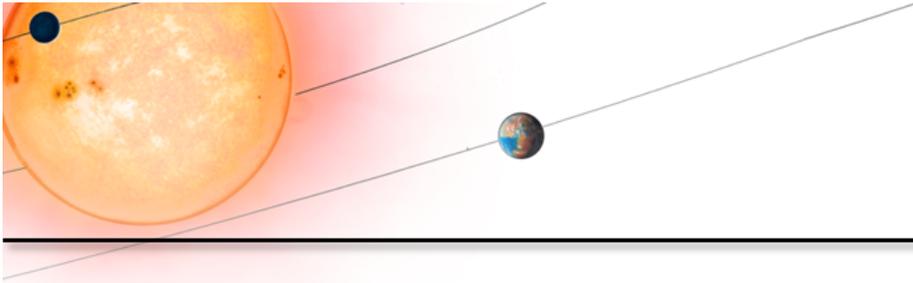
Oscillation spectra of subgiants

- Detection of mixed modes during the subgiant phase depends on the presence of a convective core during the main-sequence
 - For $1 M_{\odot}$ -star (radiative core on the MS), apparition of 1st avoided crossing in the observed frequency range $\Delta\tau = 2.2 \text{ Gyr}$ after end of the main sequence
 - $\Rightarrow \Delta\tau/\tau_{\text{MS}} = 24\%$
 - For $1.3 M_{\odot}$ -star (convective core on the MS), apparition of 1st avoided crossing in the observed frequency range $\Delta\tau = 20 \text{ Myr}$ after end of the main sequence
 - $\Rightarrow \Delta\tau/\tau_{\text{MS}} = 0.5\%$



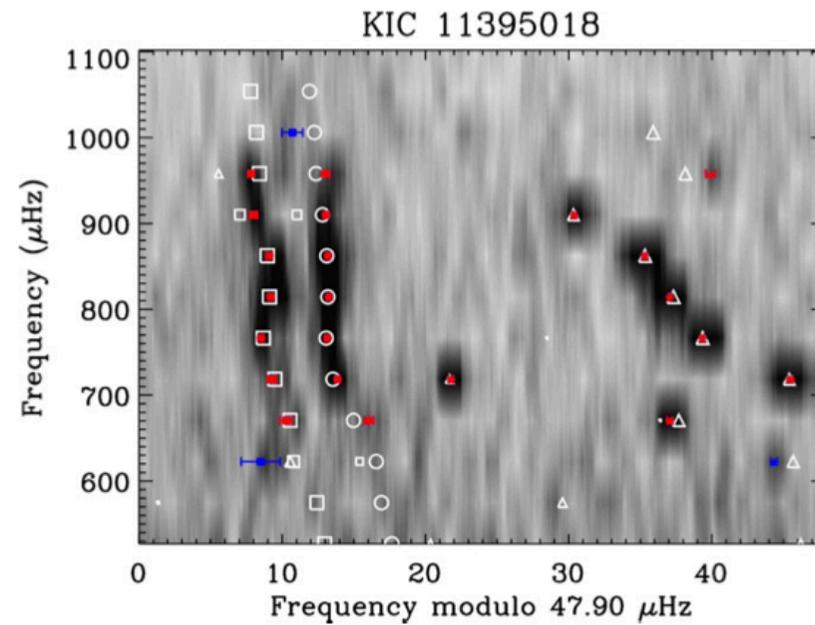
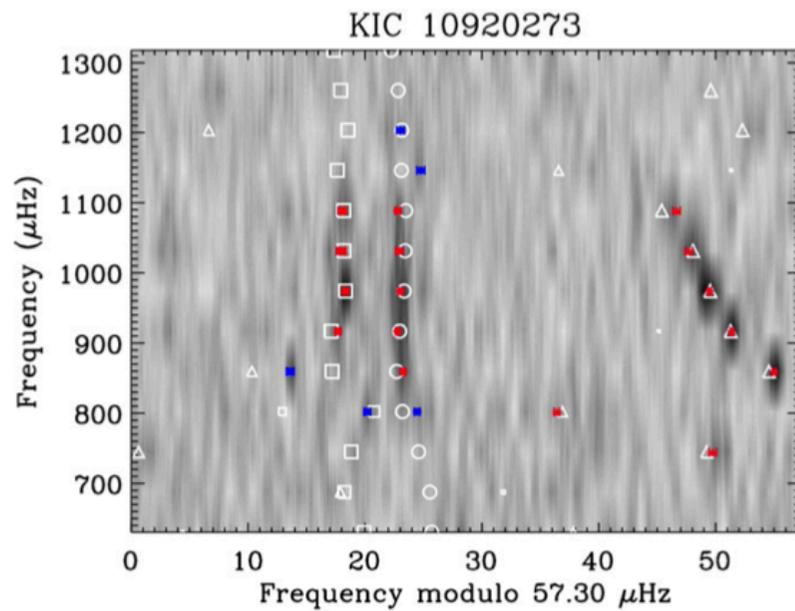
Modeling techniques

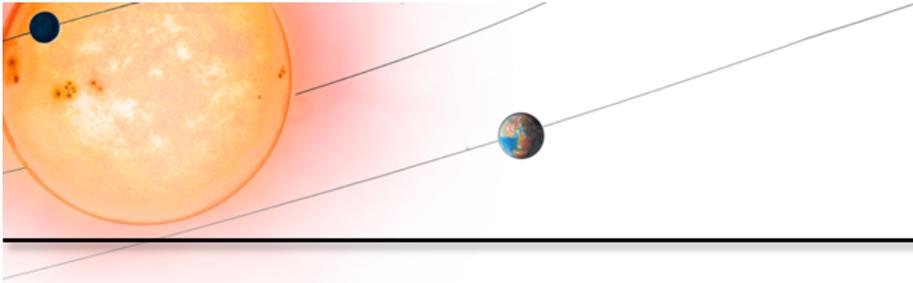
- Grid-based modeling (**Brandão et al. 2011**, **Dogan et al 2013**, **Stockholm et al. in prep.**)
 - Usually involve computation of several sub-grids with refined mesh in the regions of minimal χ^2 values
- AMP (Asteroseismic Modeling Portal, **Metcalf et al. 2009**, **Dogan et al. 2013**, **Metcalf et al. 2014**):
 - Parallel genetic algorithm for a first global search
 - Levenberg-Marquardt algorithm + Single-Value Decomposition for a local search
- Levenberg-Marquardt algorithm (**Deheuvels & Michel 2011**)
 - Adapted to treat subgiants with mixed modes
- Downhill simplex method (**Ball et al. 2017**)
 - First search within a coarse grid before applying simplex method



Performances

- Example 1: Seismic modeling of two Kepler subgiants (Doğan et al. 2013)





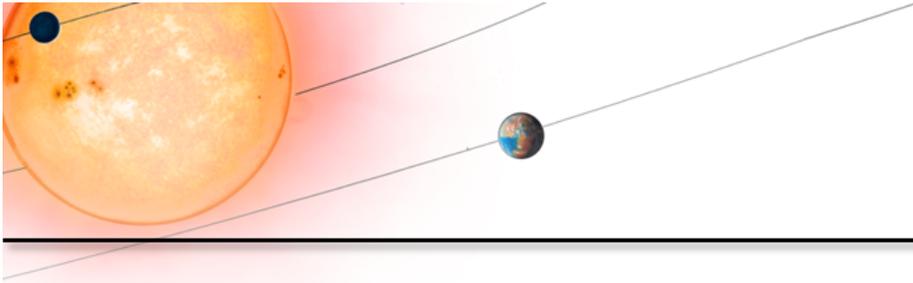
Performances

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Table 5
Fitted Parameters for KIC 11395018

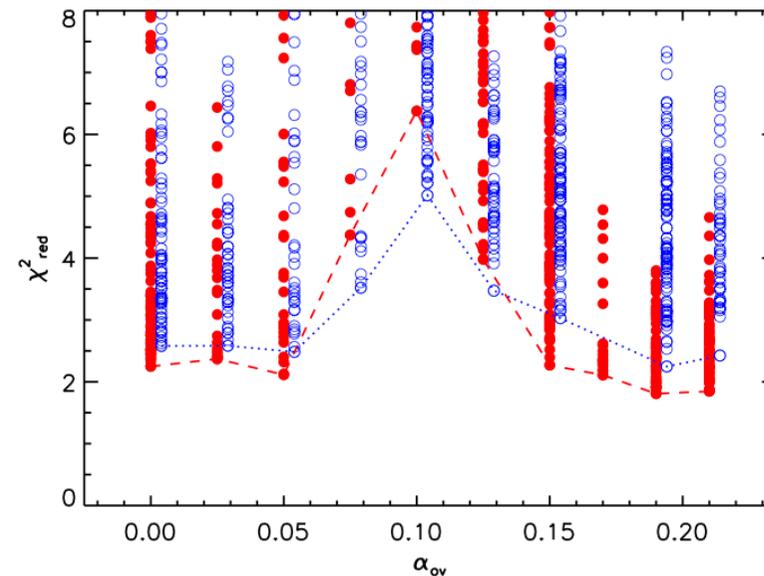
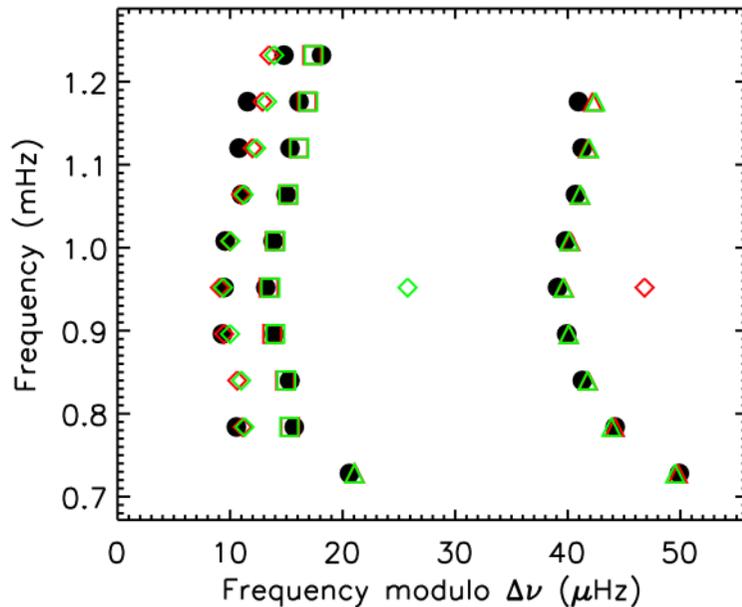
Model	M/M_{\odot}	$(Z/X)_i$	Y_i	α	t (Gyr)	L/L_{\odot}	R/R_{\odot}	T_{eff} (K)	$\log g$	[Fe/H]	X_c	χ^2_{seis}	χ^2_{atm}
BA1 (AMP)	1.23	0.034	0.301	1.94	4.46	4.40	2.158	5697	3.860	0.144	0.0	8.07	0.49
BA2 (AMP)	1.32	0.028	0.241	1.94	4.84	4.53	2.210	5671	3.869	0.049	0.0	8.51	0.69
BA3 (AMP) ^a	1.26	0.034	0.294	2.02	4.28	4.64	2.175	5749	3.863	0.140	0.0	7.36	0.55
BB1 (ASTECC1)	1.235	0.033	0.282	1.80	5.05	4.04	2.156	5573	3.861	0.114	0.0	17.03	1.02
BB2 (ASTECC1) ^a	1.22	0.040	0.310	1.80	4.65	4.11	2.154	5605	3.858	0.213	0.0	17.08	1.02
BC1 (ASTECC2)	1.25	0.034	0.297	1.81	4.29	4.42	2.265	5615	3.820	0.116	0.0	47.34	0.90
BC2 (ASTECC2) ^b	1.32	0.043	0.270	1.81	4.89	4.06	2.211	5515	3.870	0.243	0.0	17.65	2.01
BD (CESAM)	1.29	0.030	0.260	0.64 ^c	4.50	4.89	2.190	5806	3.866	0.026	0.0	4.04	1.19
BE (Geneva)	1.32	0.033	0.275	1.80	4.30	5.18	2.207	5867	3.871	0.120	0.0	115.69	1.37
Weighted mean	1.27	0.033	0.276		4.57	4.54	2.184	5706	3.863	0.103			
Standard deviation	(0.04)	(0.004)	(0.022)		(0.23)	(0.30)	(0.024)	(92)	(0.008)	(0.070)			
Creevey et al. (2012)	1.37 ± 0.11				3.9 ± 1.4 4.5 ± 0.5^d	4.2 ± 1.1	2.23 ± 0.04		3.88 ± 0.02 3.86 ± 0.03^c				

- Large values of χ^2_{red} for best-fit models (even with 8 mths of data)
 - Caused by problems in stellar models ? Or pb in the fitting procedure...
- Large disparities in the reduced χ^2 of the best-fit models for different modeling techniques!

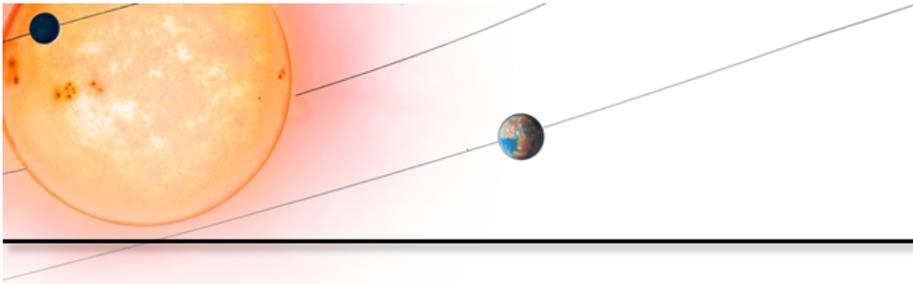


Performances

- Example 2: Seismic modeling of a CoRoT subgiant (Deheuvels & Michel 2011)

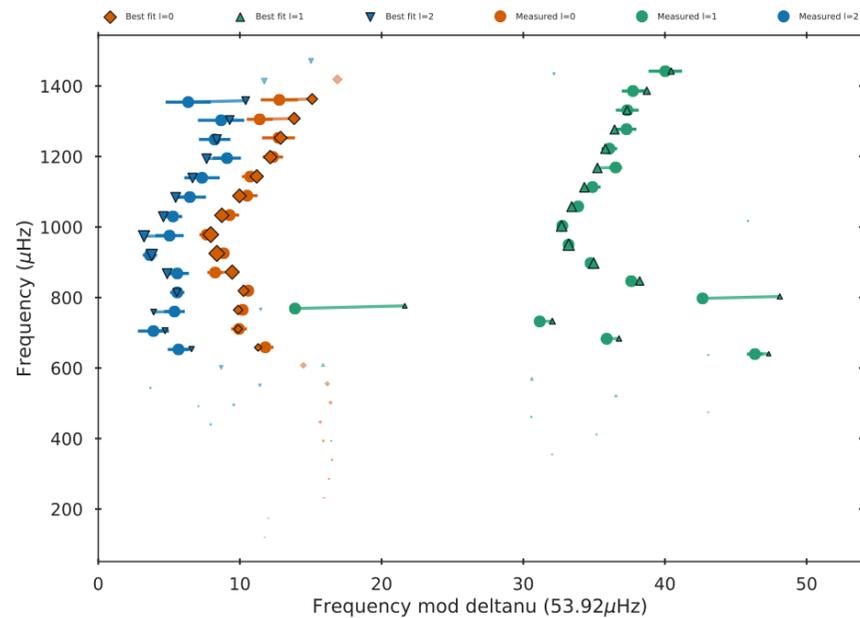


- Good statistical agreement with data (“only” 5 months of data)
- Two possible solutions, with either a small ($d_{ov} < 0.05 H_p$) or a large ($d_{ov} \sim 0.20 H_p$) amount of core overshooting during the MS

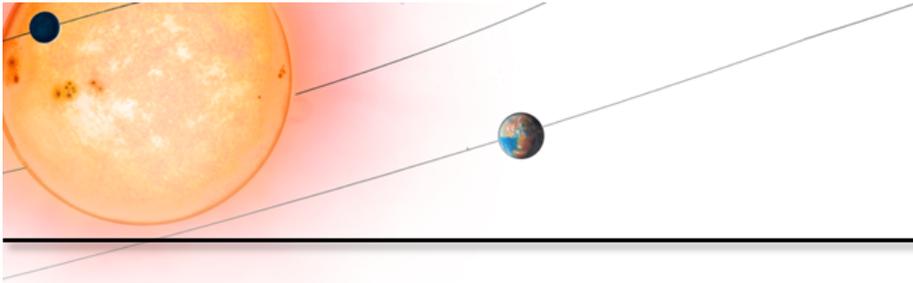


Performances

- Example 3: Seismic modeling of a bright Kepler subgiant with BASTA (Stockholm et al. in prep)

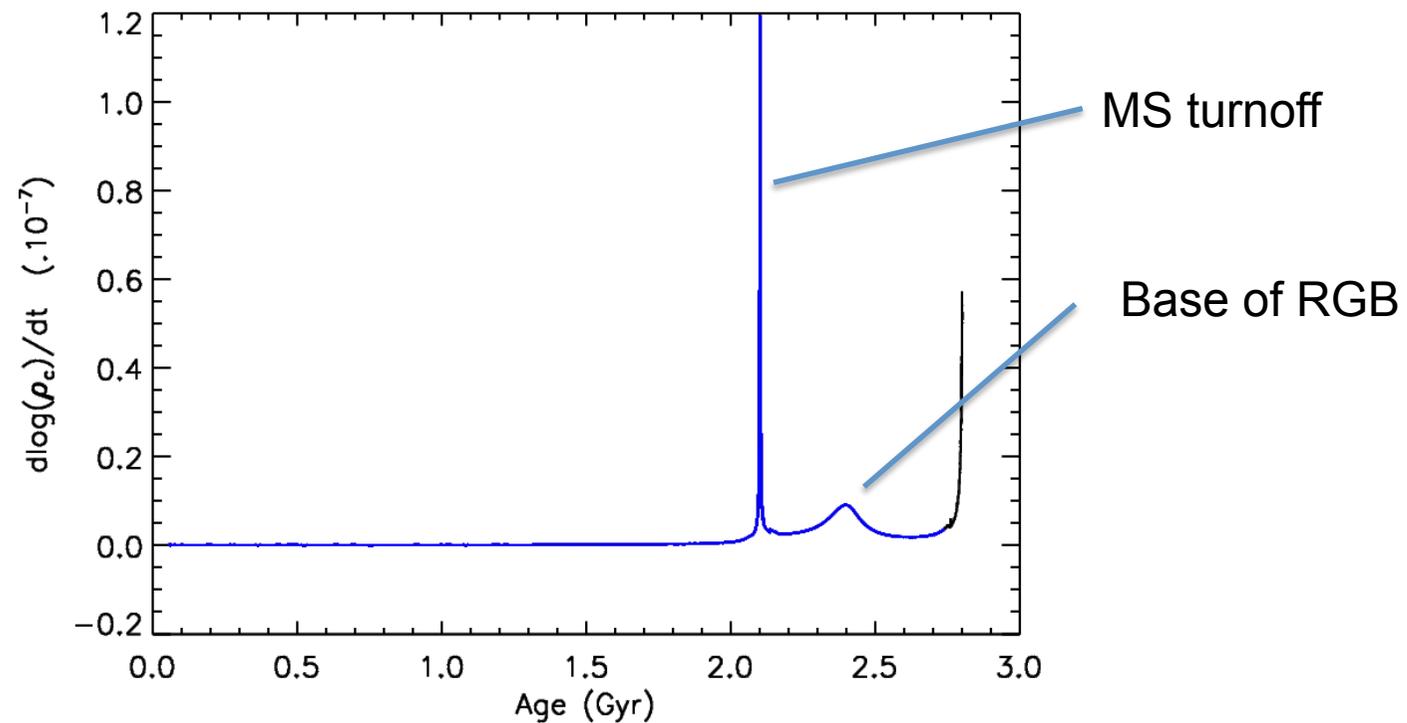


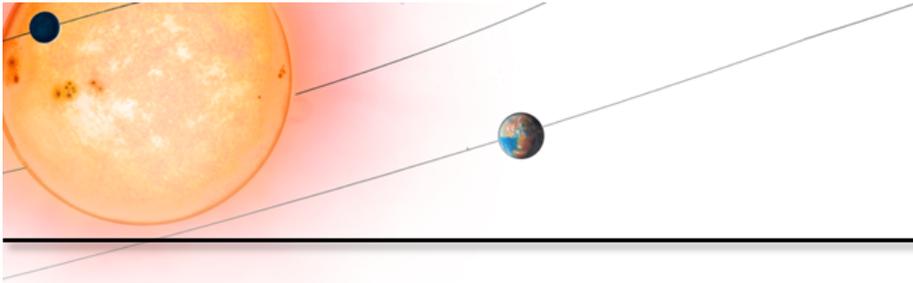
- Grid details: $\Delta M = 0.01 M$, $\Delta(Z/X) = 0.03$ dex



Sensitivity of mixed modes to **stellar age**

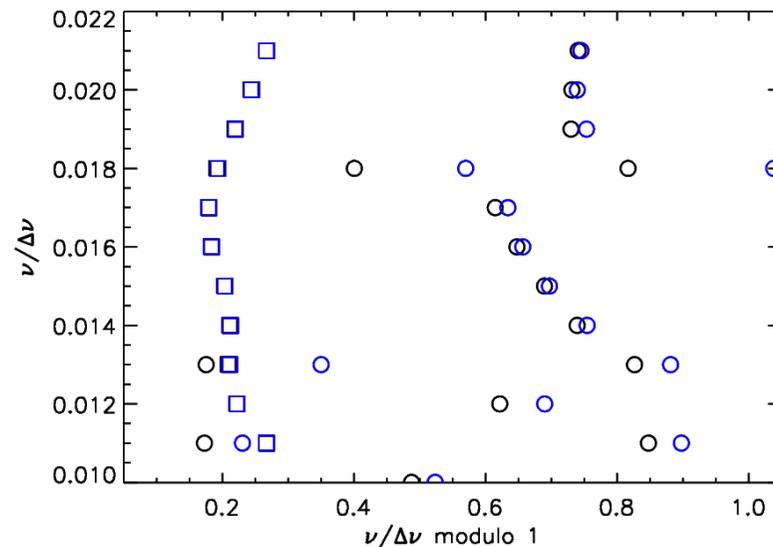
- Variations in the frequencies of g modes tightly related to those of ρ_c



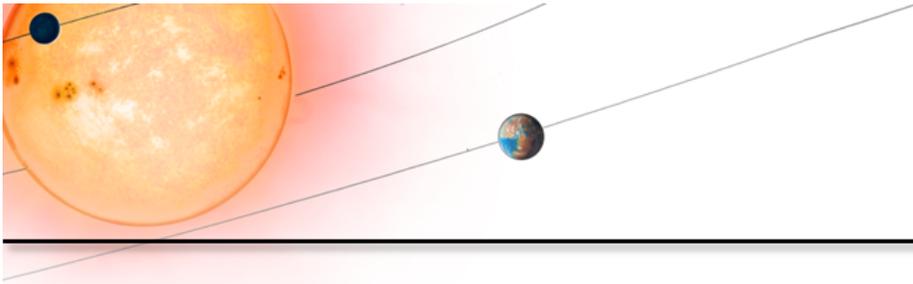


Sensitivity of mixed modes to **stellar age**

- Variations in the normalized frequencies after $\Delta t = 10$ Myr



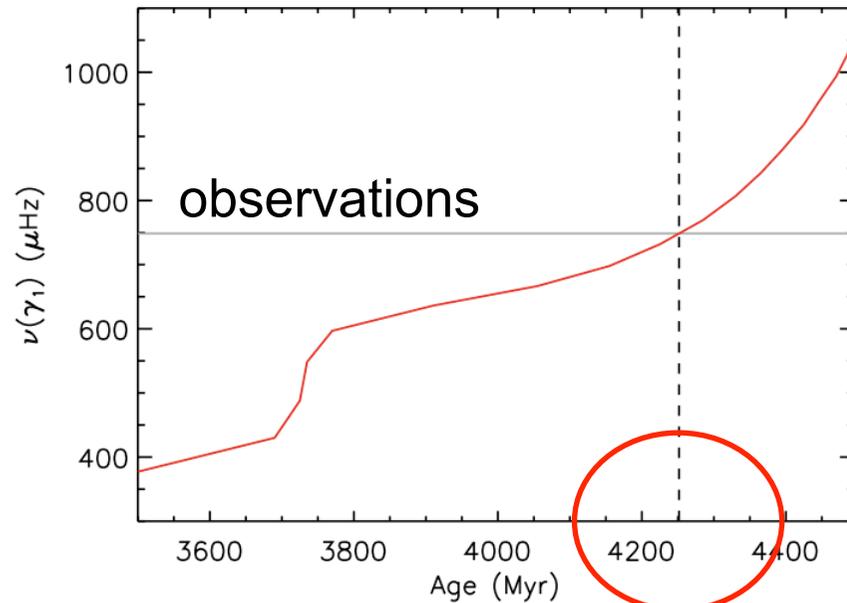
- Typically $\dot{\nu}_g \sim 1 \mu\text{Hz}.\text{Myr}^{-1}$ \Rightarrow requires grid-step ~ 1 Myr
- The rate of variations in the frequencies of mixed modes also changes rapidly: for p-dominated modes, $\dot{\nu}_{\text{mix}} < 0$ while for g-dominated modes, $\dot{\nu}_{\text{mix}} > 0$



Sensitivity of mixed modes to **stellar age**

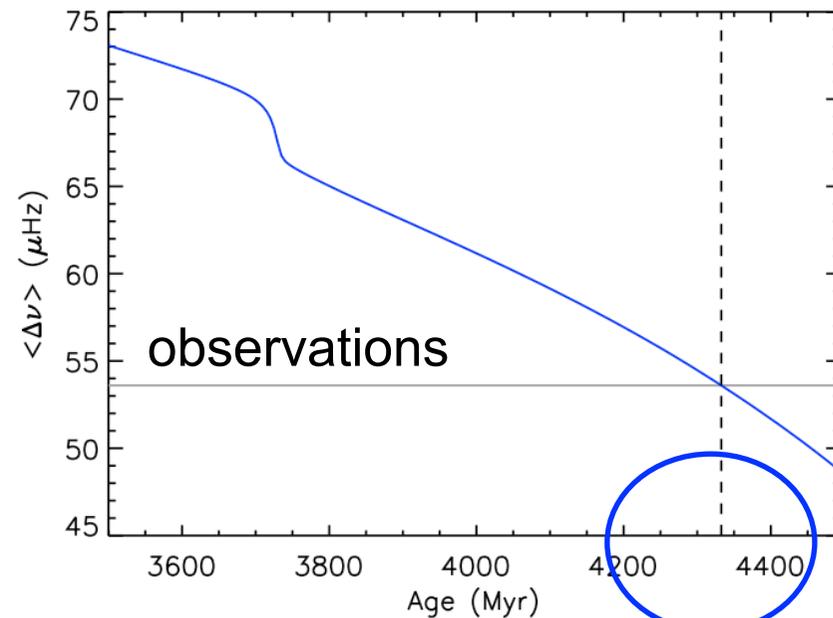
g-mode frequencies

$$\nu_g \propto \rho_c$$



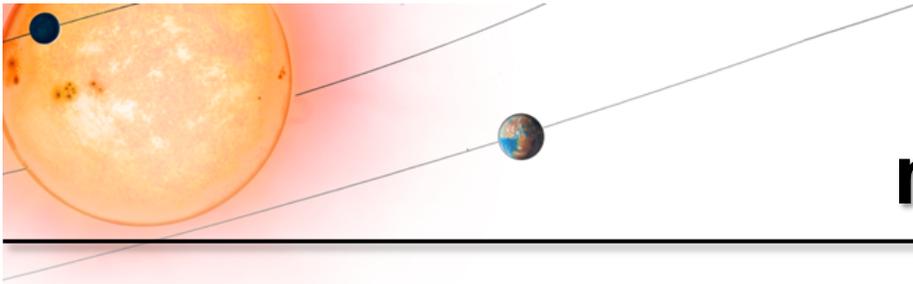
large separation

$$\langle \Delta\nu \rangle \propto \frac{GM}{R^3}$$



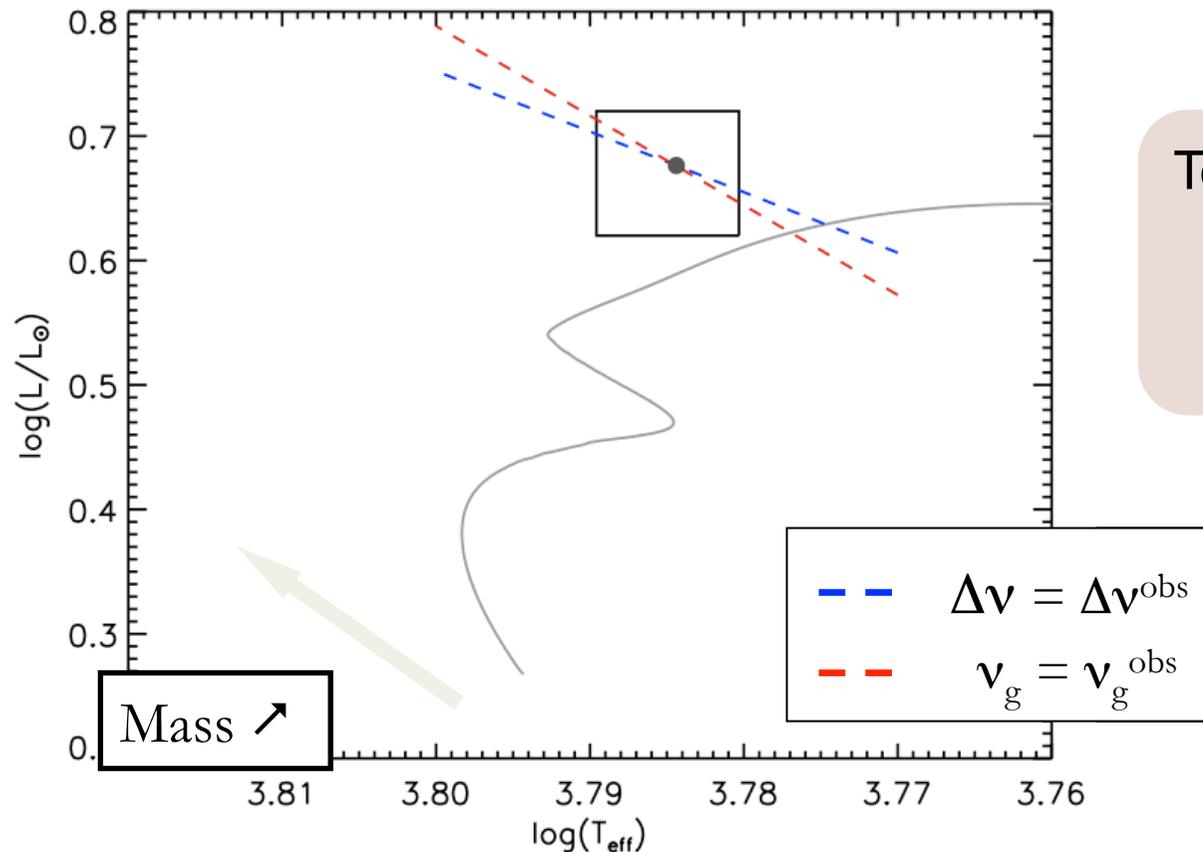
ages a priori different

Which are the models that fit both conditions simultaneously?



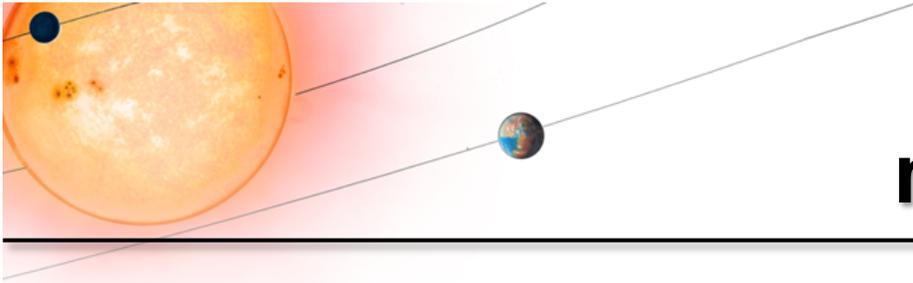
Sensitivity of mixed modes to **stellar mass**

- We showed that for a given physics, the knowledge of $\Delta\nu$ and ν_g imposes one and one only mass (Deheuvels & Michel 2011)



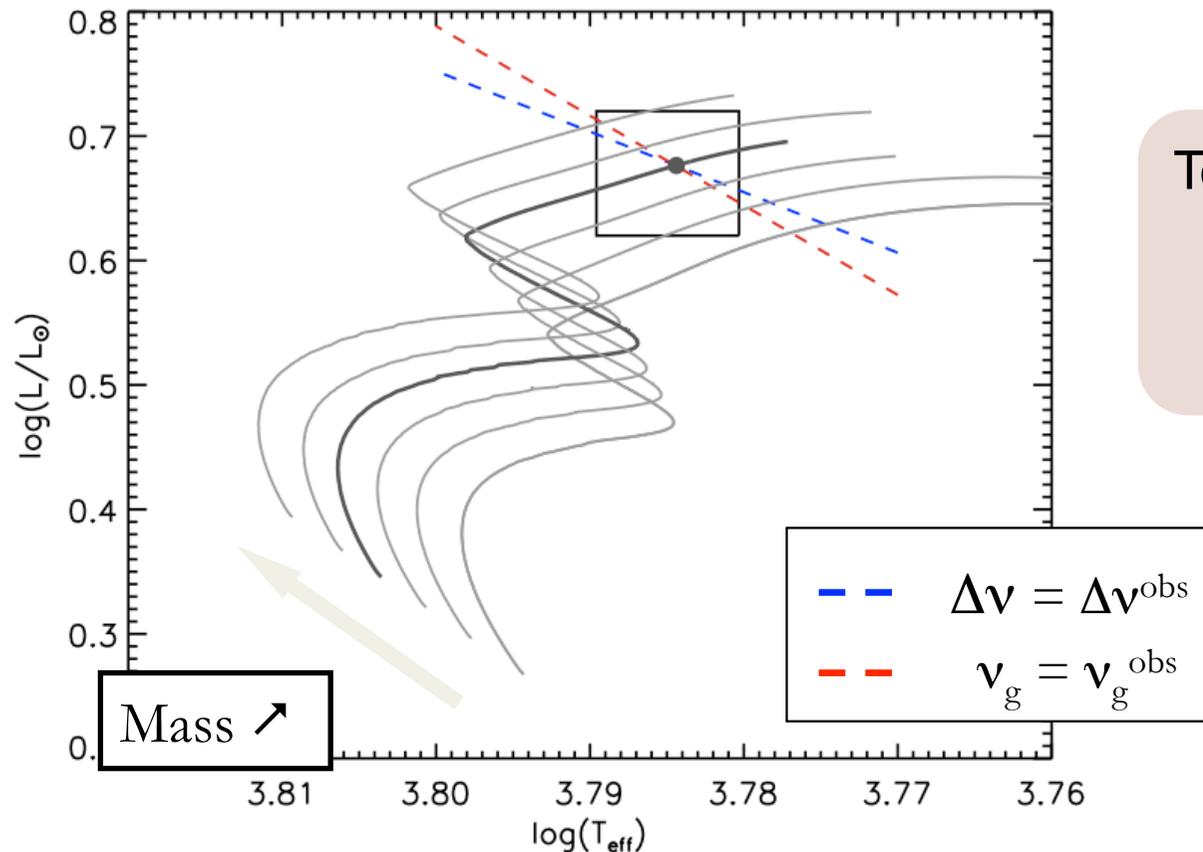
To verify both conditions

- one mass only (\tilde{M})
- one age only ($\tilde{\tau}$)



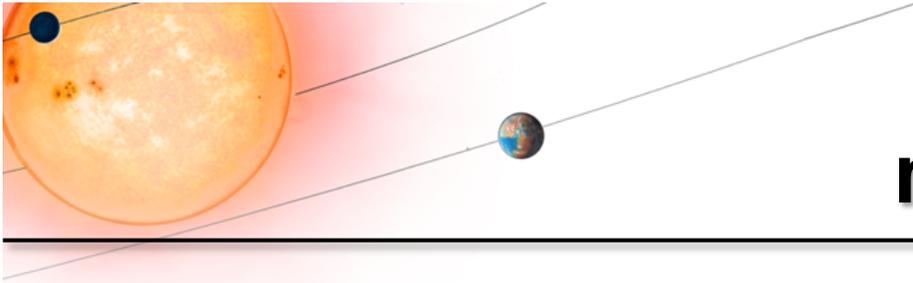
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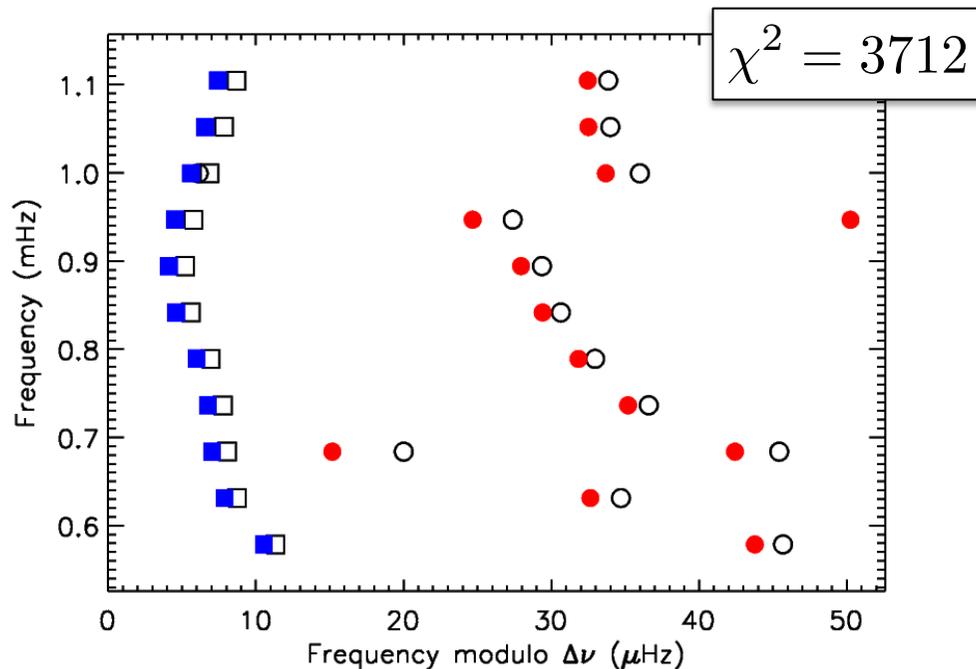
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Sensitivity of mixed modes to **stellar mass**

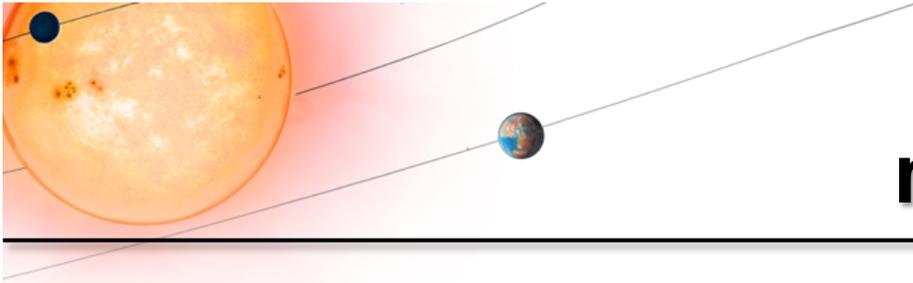
- Question: *What grid-mesh is required for the stellar mass to model subgiants?*
- Grid with time step of 1 Myr, step of $\Delta M = 0.01 M_{\odot}$ in mass ($M = 1.29, 1.30, 1.31 M_{\odot}$), fixed input physics
- Reference model: $M = 1.295 M_{\odot}$, Age = 4200 Myr, same physics as grid



$$\chi^2 = \sum \frac{(\nu_i^{\text{mod}} - \nu_i^{\text{ref}})^2}{\sigma_i^2}$$

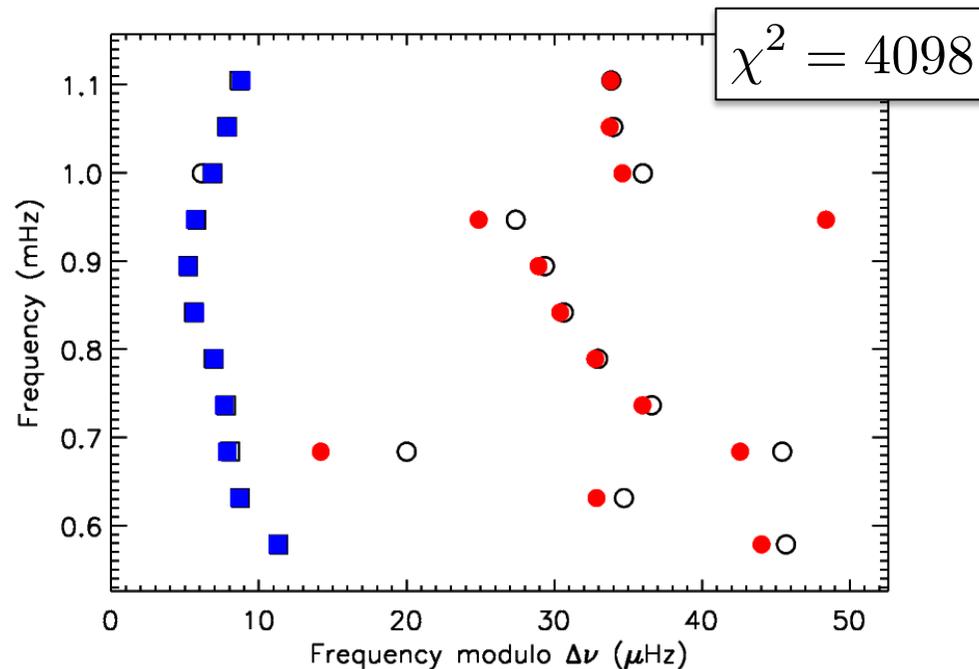
- We assume $\sigma_{\nu} = 0.2 \mu\text{Hz}$
- With a perfect model, we expect

$$E(\chi^2) = n_{\text{freq}} = 24$$



Sensitivity of mixed modes to **stellar mass**

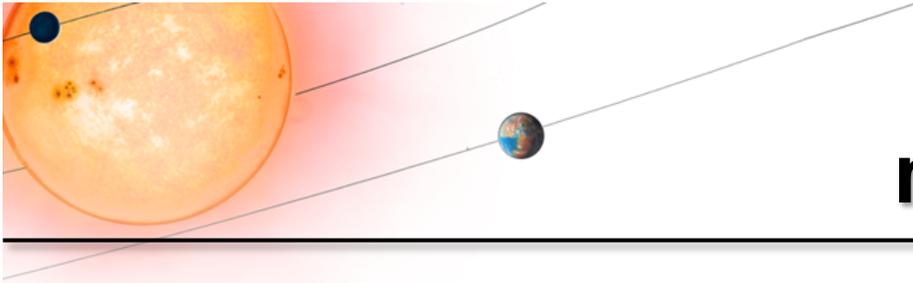
- Question: *What grid-mesh is required for the stellar mass to model subgiants?*
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$$\chi^2 = \sum \frac{(\nu_i^{\text{mod}} - \nu_i^{\text{ref}})^2}{\sigma_i^2}$$

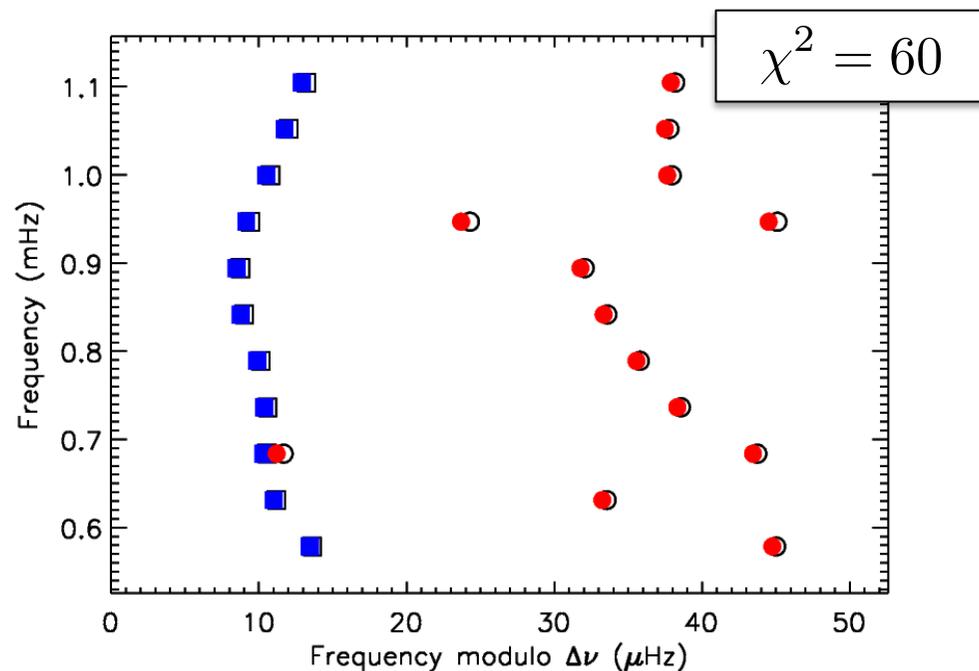
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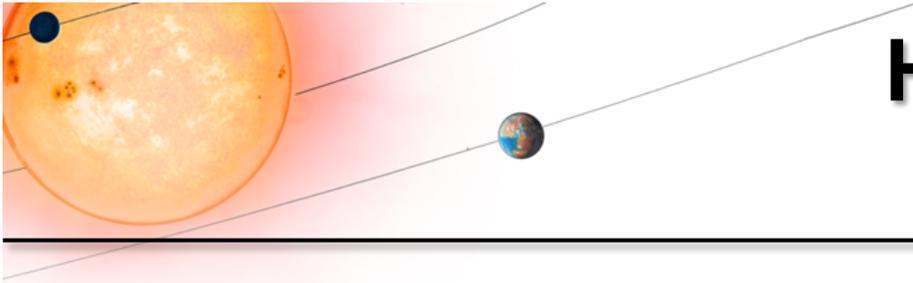


Sensitivity of mixed modes to **stellar mass**

- Question: *What grid-mesh is required for the stellar mass to model subgiants?*
- Grid with time step of 1 Myr, step of $\Delta M = 0.001 M_{\odot}$ in mass ($M = 1.299, 1.300, 1.301 M_{\odot}$), fixed input physics
- Reference model: $M = 1.2995 M_{\odot}$, Age = 4110 Myr, same physics as grid

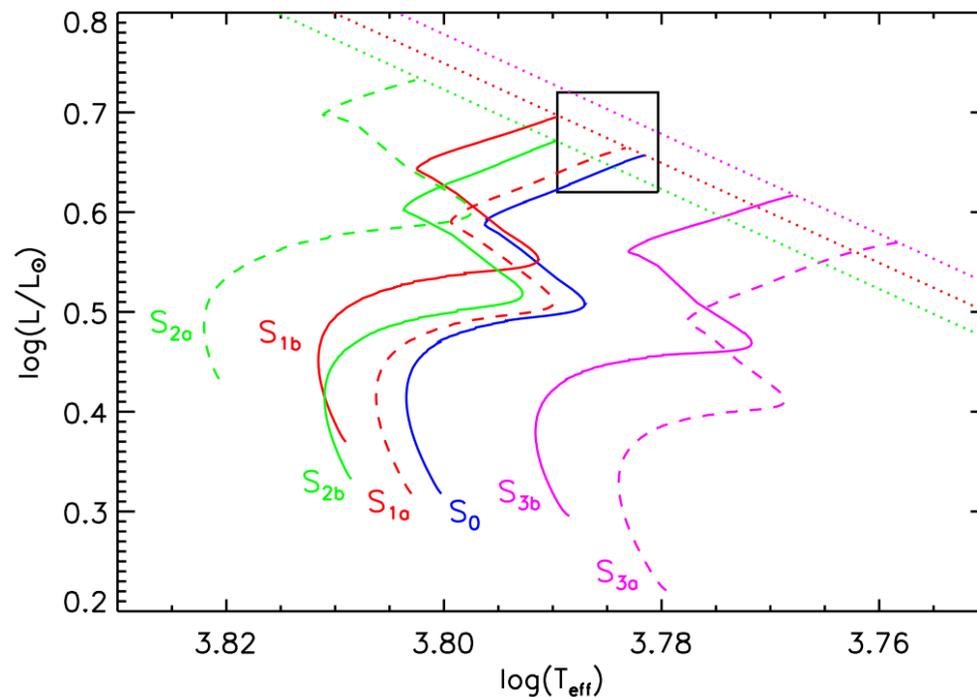


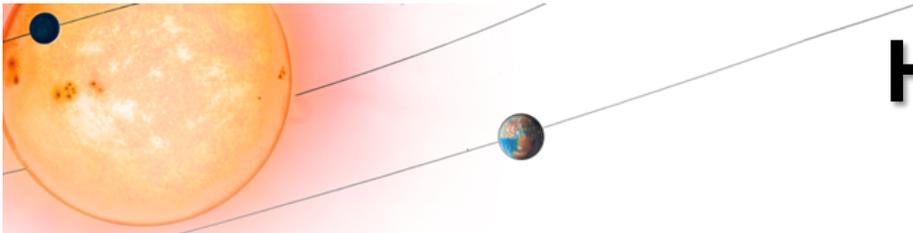
- With a precision of $\sigma_v = 0.2 \mu\text{Hz}$, a **grid mesh $\Delta M < 0.001 M_{\odot}$ is required in mass to avoid “missing” models**



How does \tilde{M} vary with stellar parameters?

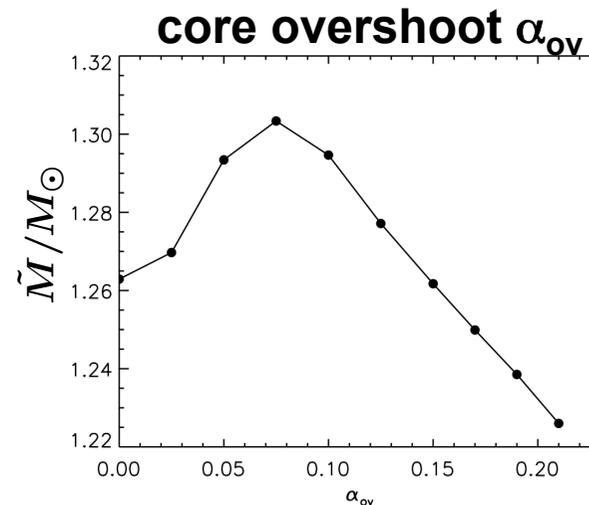
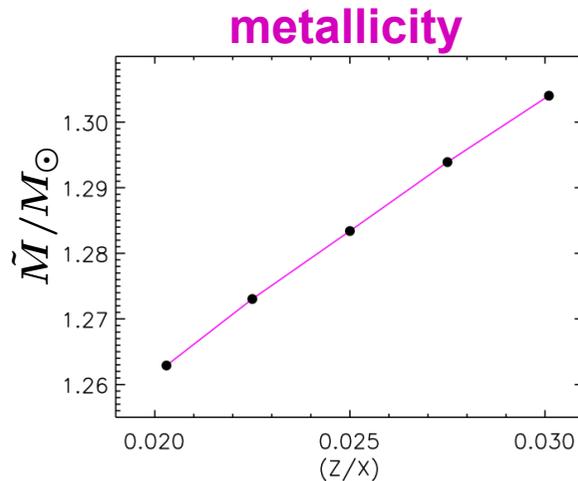
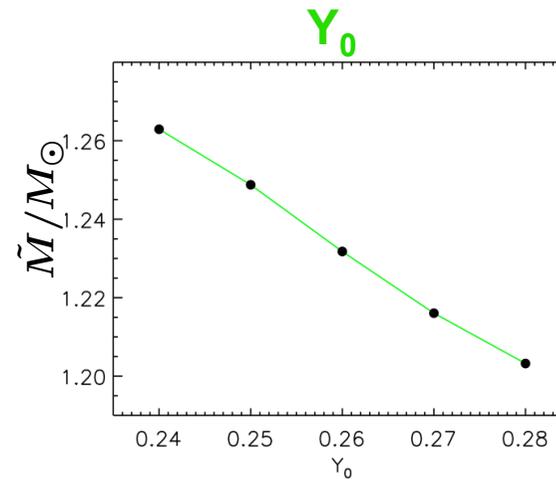
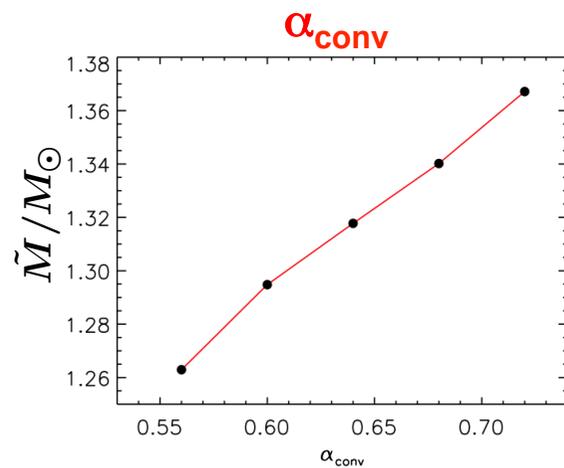
- Idea: using the observed value of $\Delta\nu$ and the frequency of the most g-like mode(s) to pre-select models
- Variations in \tilde{M} for varying **mixing length parameter** α_{conv} , **initial helium** Y_0 , and **metallicity** (AGS05 to GN93)

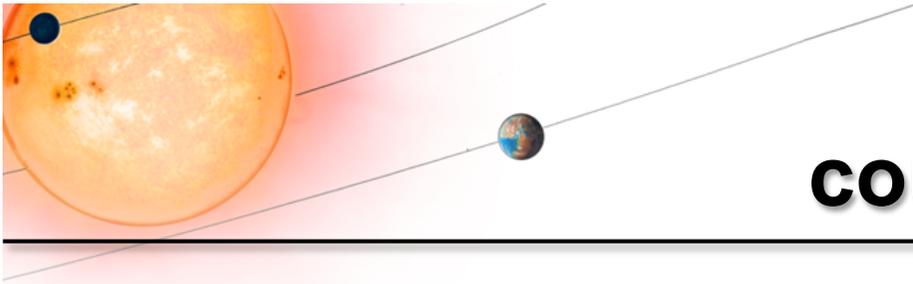




How does \tilde{M} vary with stellar parameters?

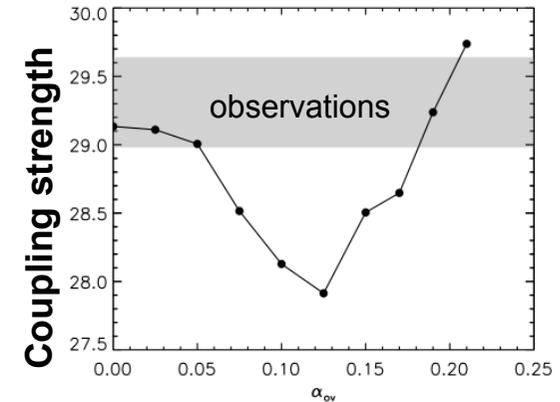
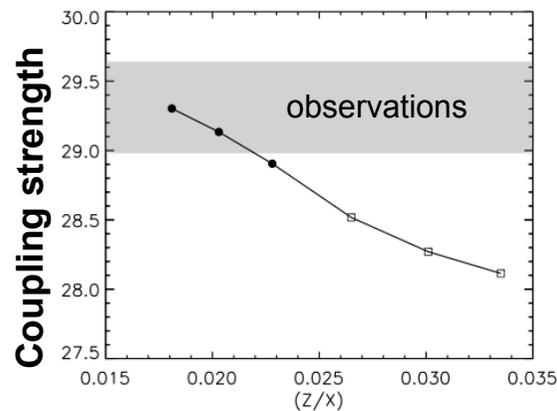
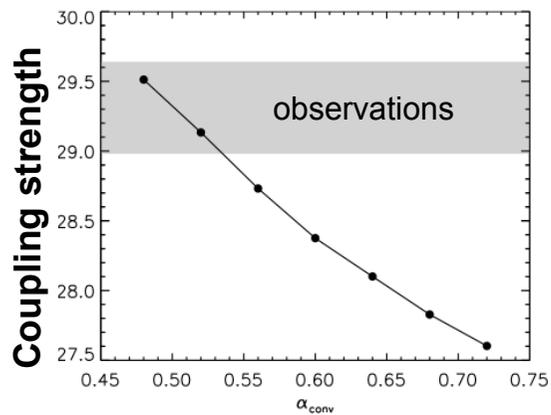
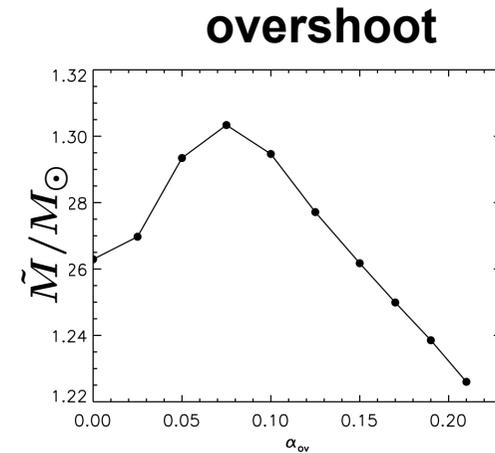
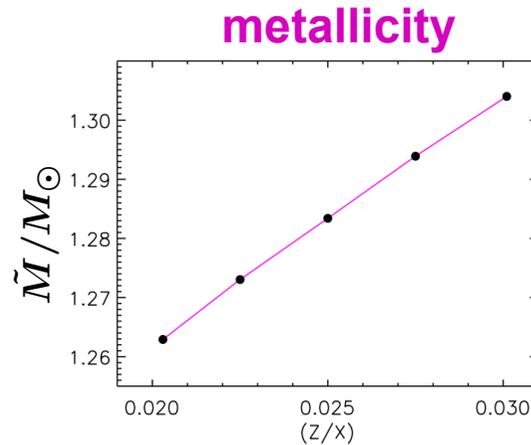
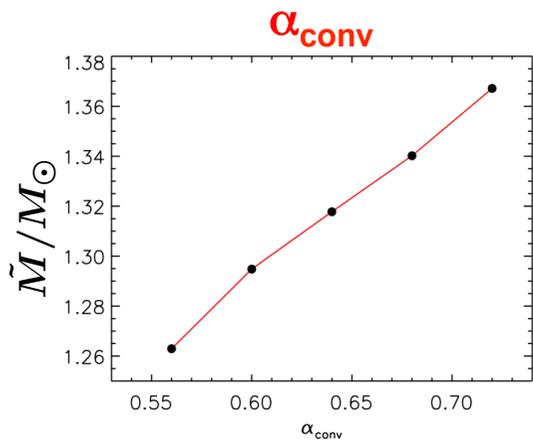
- Variations in \tilde{M} for varying **mixing length parameter** α_{conv} , **initial helium** Y_0 , **metallicity**, and **core overshoot** α_{ov}

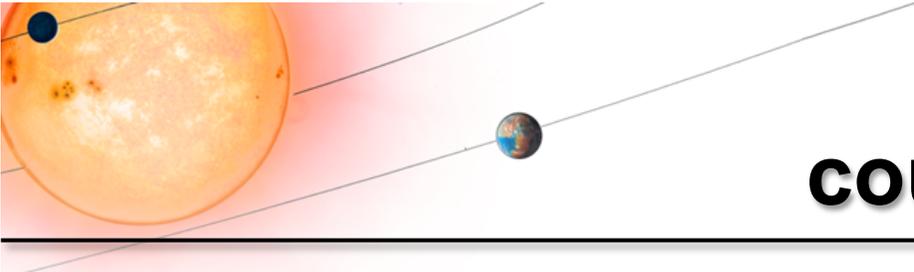




Link between mode coupling and stellar mass

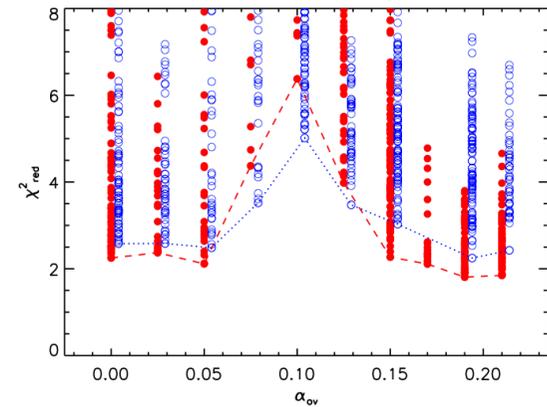
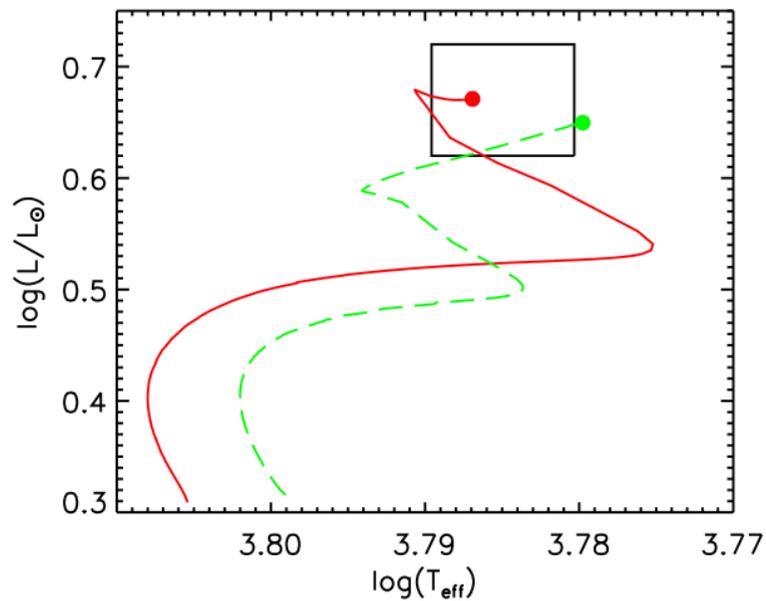
- Seismic modeling of HD49385 (Deheuvels & Michel 2011)
 - Tight anti-correlation between mass and coupling strength

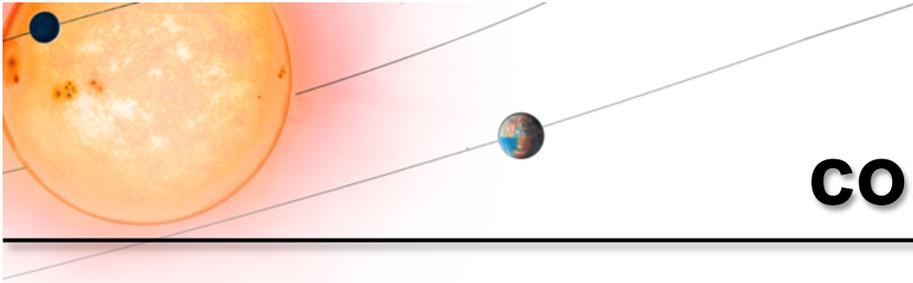




Link between mode coupling and stellar mass

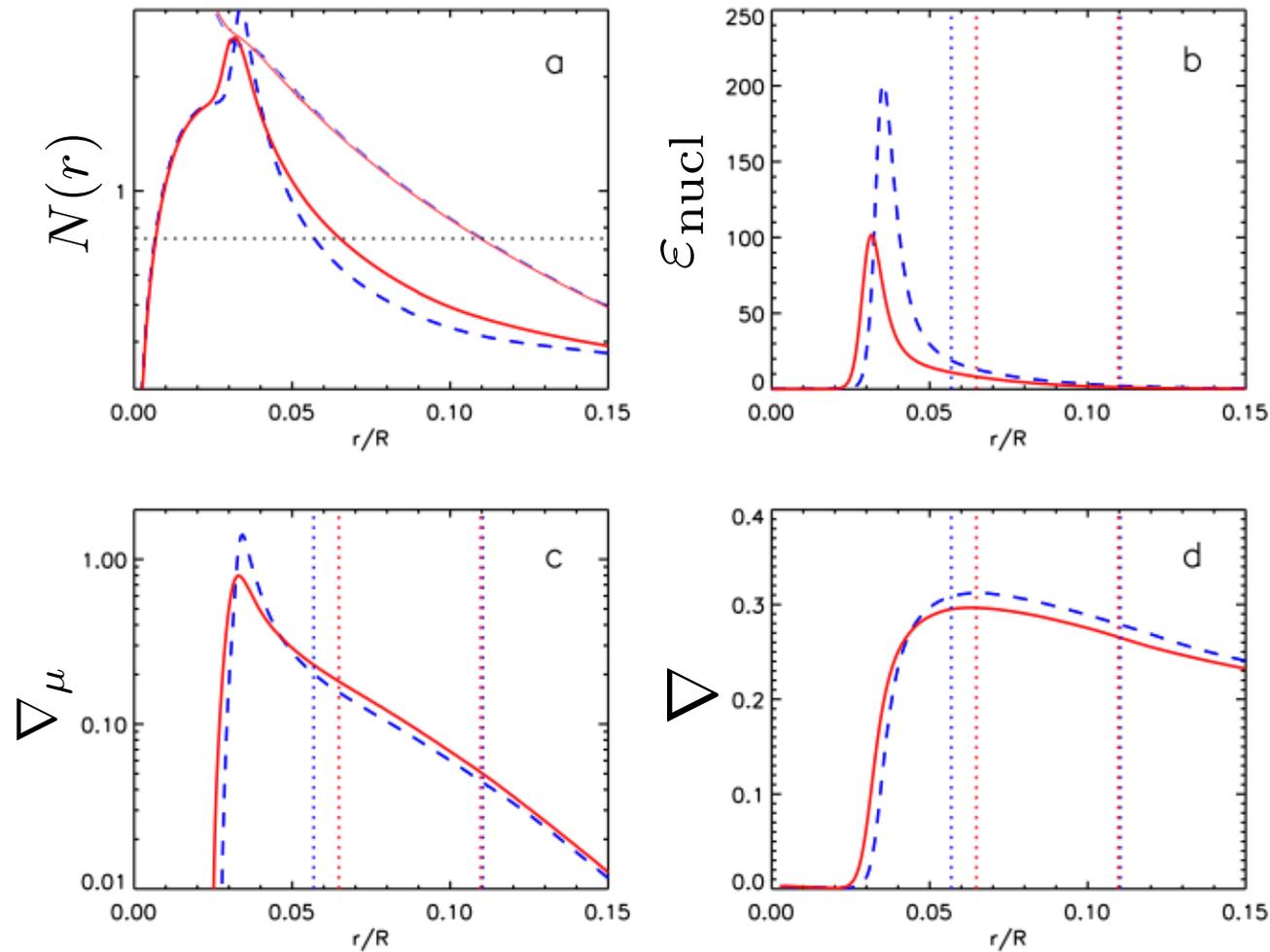
- Seismic modeling of HD49385 (Deheuvels & Michel 2011)
 - Tight anti-correlation between mass and coupling strength
 - Reason for bimodality of solutions for this target
 - Not specific to this target, expected to occur for stars close to the turnoff

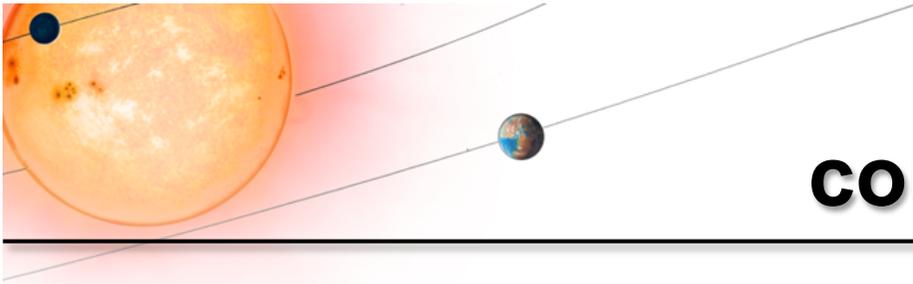




Link between mode coupling and stellar mass

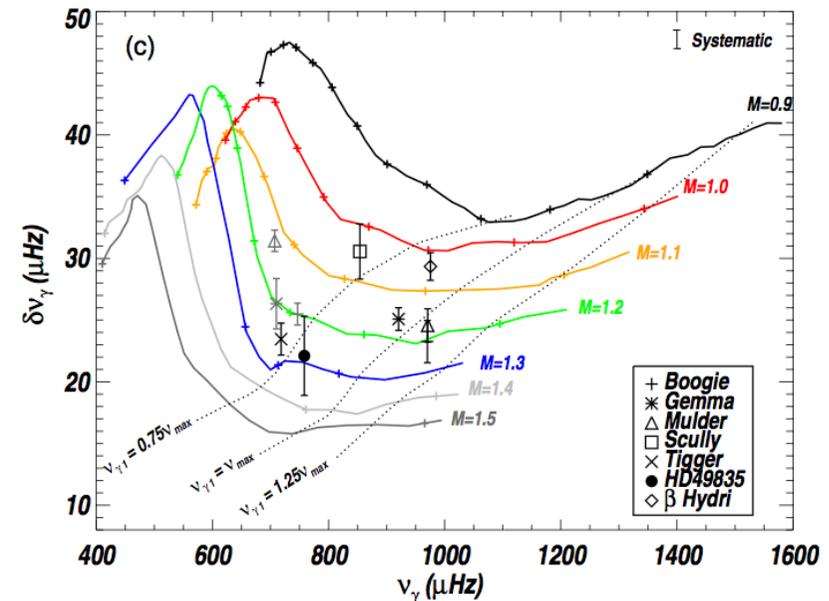
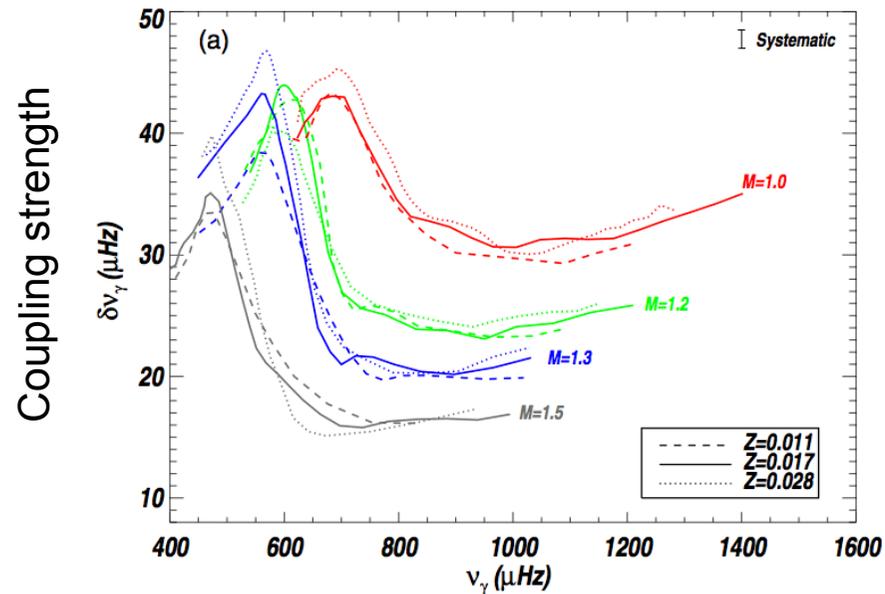
- Anti-correlation between mass and coupling strength (Deheuvels & Michel 2011)

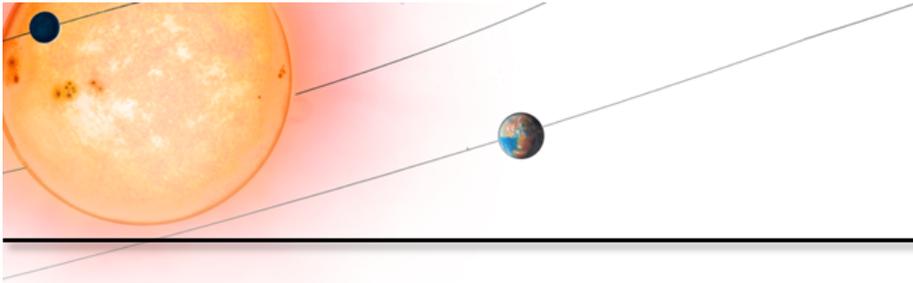




Link between mode coupling and stellar mass

- Anti-correlation btw mass and coupling strength also found by fitting asymptotic expressions to mode frequencies of stellar models (Benomar et al. 2012)
 - Proposed as a way of estimating stellar masses





Correction of near-surface effects

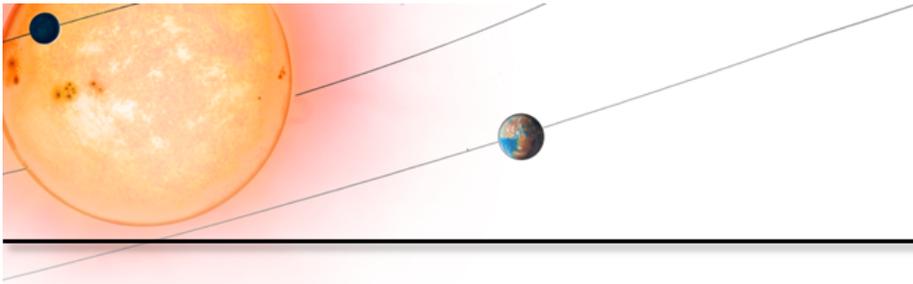
- Issue of the correction of near-surface effects
 - Using frequency ratios such as r_{01} or r_{10} is impractical because of mixed modes
 - 3D simulations of convection are far too computationally demanding for stellar modeling
 - Only empirical corrections are thus available so far (e.g., [Kjeldsen et al. 2008](#), [Ball & Gizon 2014](#), [Sonoji et al. 2015](#))

$$\nu_{n,\ell}^{\text{obs}} - \nu_{n,\ell}^{\text{best}} = a \left(\frac{\nu_{n,\ell}^{\text{obs}}}{\nu_0} \right)^b$$

$$\nu_{\text{cor}} - \nu_{\text{mdl}} = \frac{s_0 \nu_{\text{max}}}{Q} \left[1 - \frac{1}{1 + (\nu_{\text{mdl}}/\nu_{\text{max}})^{s_1}} \right]$$

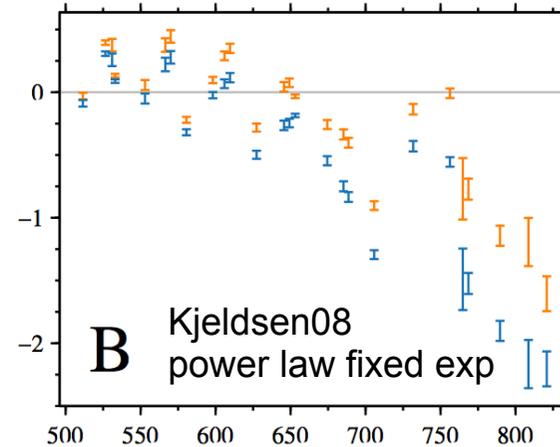
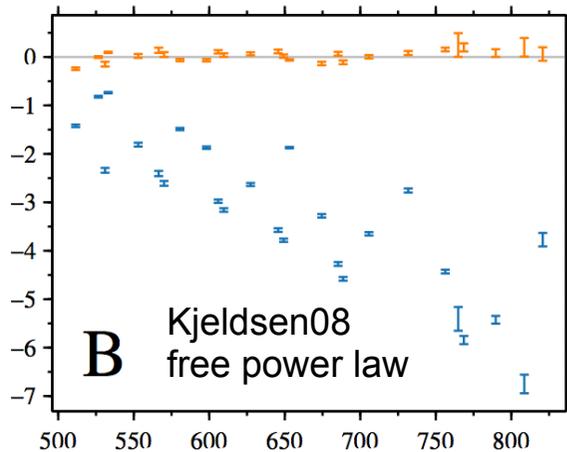
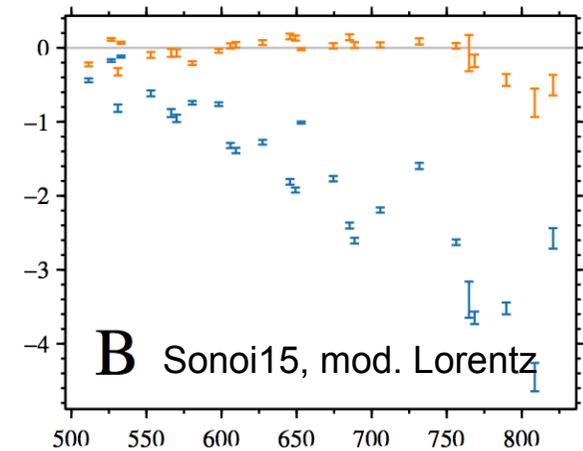
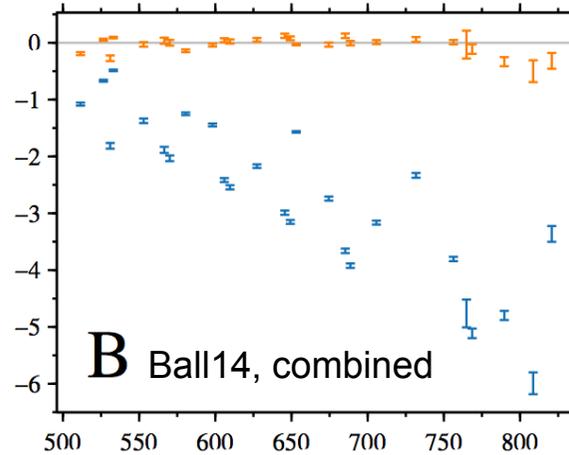
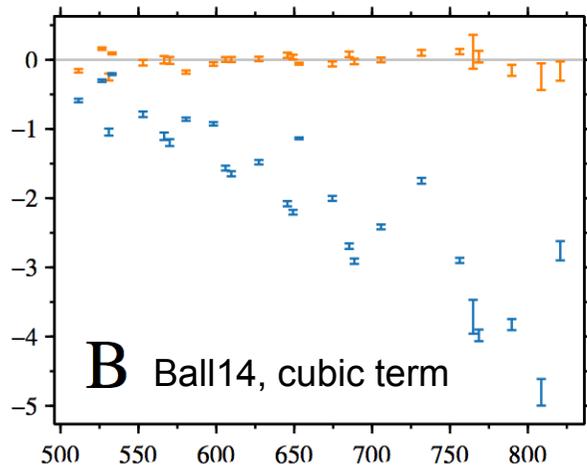
$$\nu_{\text{cor}} - \nu_{\text{mdl}} = a_3 \left(\frac{\nu_{\text{mdl}}}{\nu_{\text{ac}}} \right)^3 / \mathcal{I}$$

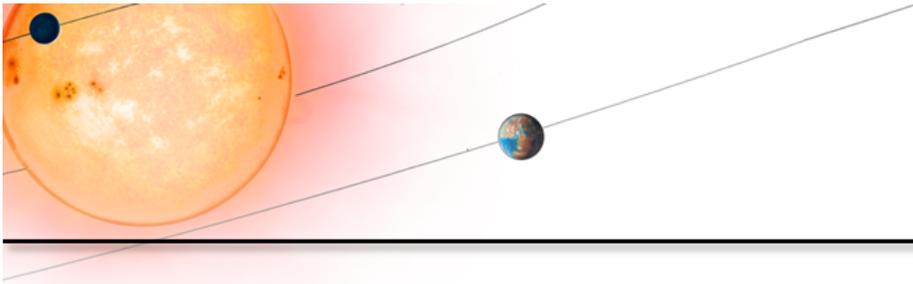
$$\nu_{\text{cor}} - \nu_{\text{mdl}} = \left[a_{-1} \left(\frac{\nu_{\text{mdl}}}{\nu_{\text{ac}}} \right)^{-1} + a_3 \left(\frac{\nu_{\text{mdl}}}{\nu_{\text{ac}}} \right)^3 \right] / \mathcal{I}$$



Correction of near-surface effects

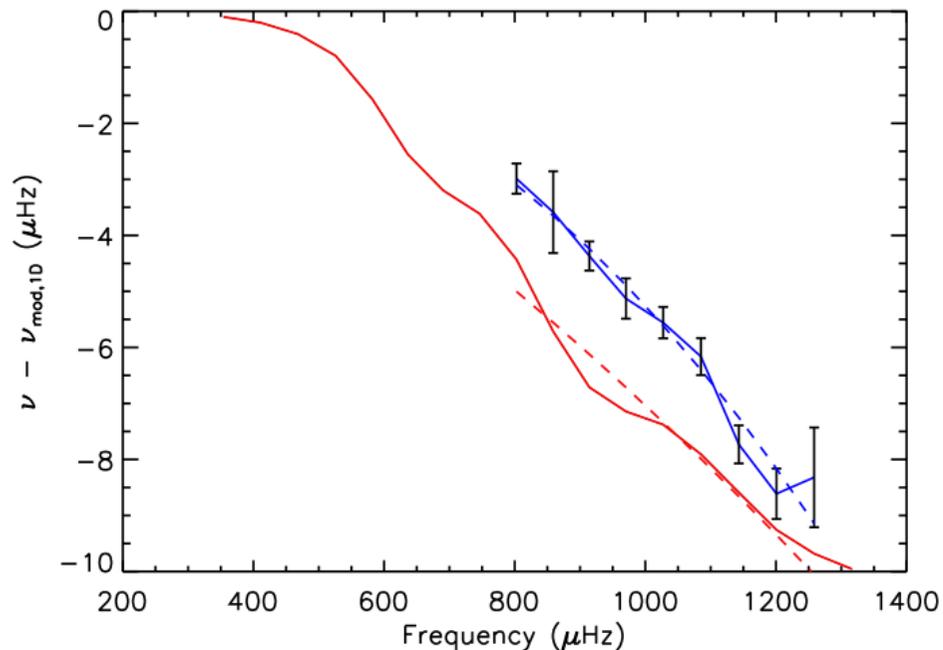
- Issue of the correction of near-surface effects (Ball et al. 2017)



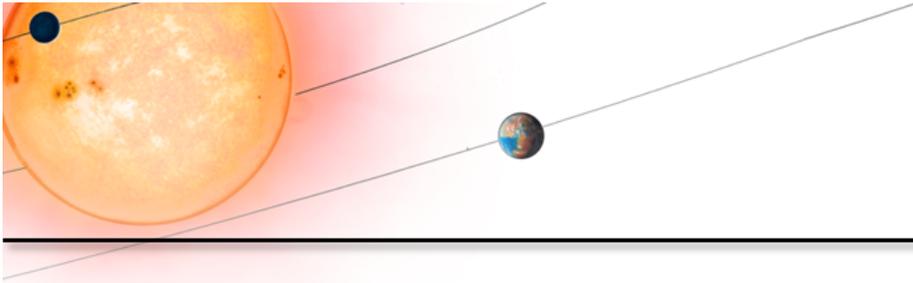


Correction of near-surface effects

- Alternative way: learn from “patched” models
- Example: 3D numerical simulation of the outer layers of CoRoT subgiant HD49385 with ANTARES (by Friedrich Kupka) “patched” to seismic 1D stellar model



	Observed NSE	Predicted NSE (patched models)
a (μHz)	-5.42	-7.18
b	2.40	1.55



Discussion

- Interest of performing interpolations within grids?
- Investigate potential of structural inversions using mixed modes